SHEAR STRESSES AND SHEAR FLOW IN BEAMS

In-Class Activities:

- Follow up
- Concepts
- Theory, formula, steps
- Applications
- Problem Solving
- Quiz types
- Exam discussion

Today's Objective : 21st September

To:

a) Understand the context, concept and derivations for shear stresses and flow

b) Be able to calculate max and specific values.

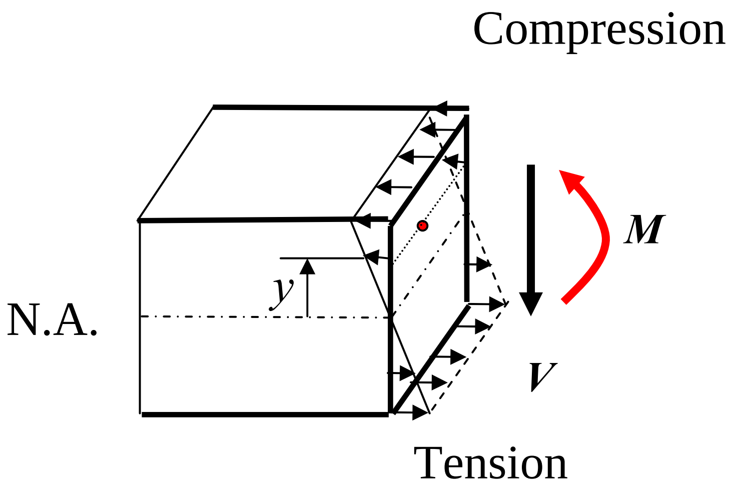
Transverse Shear in Beams

1. Concept of shear stress
2. Shear stress formula
3. Application

Revision: Bending Stress

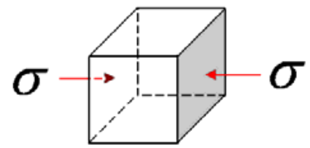
In previous lectures methods for evaluating bending moment and shear forces have been described and the equations relating bending stress to bending moment demonstrated.

$$\sigma = -\frac{My}{I}$$



Bending stresses only exist perpendicular to the beam's cross-section

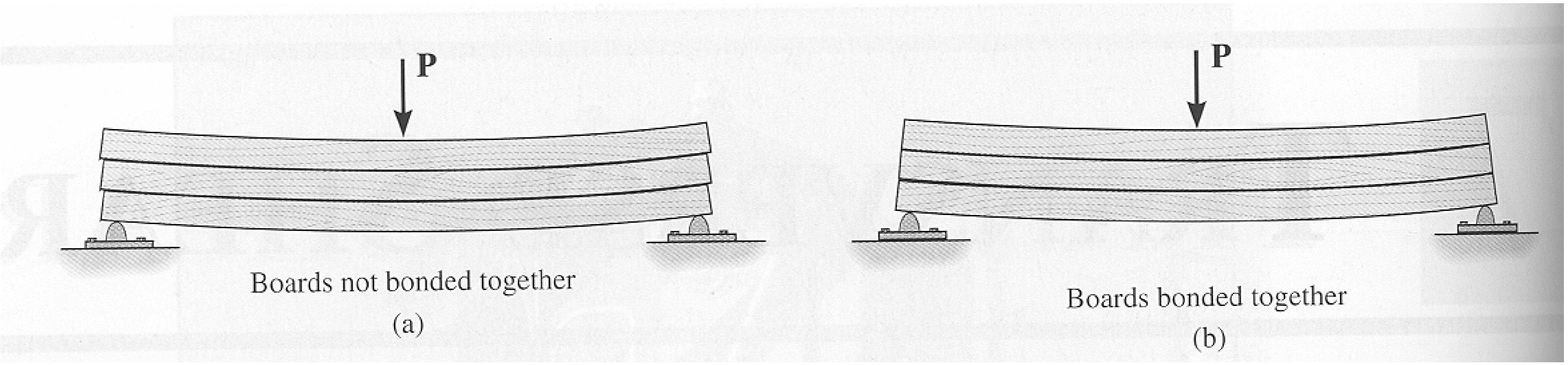
Volume element at the point:



How about the shear force?

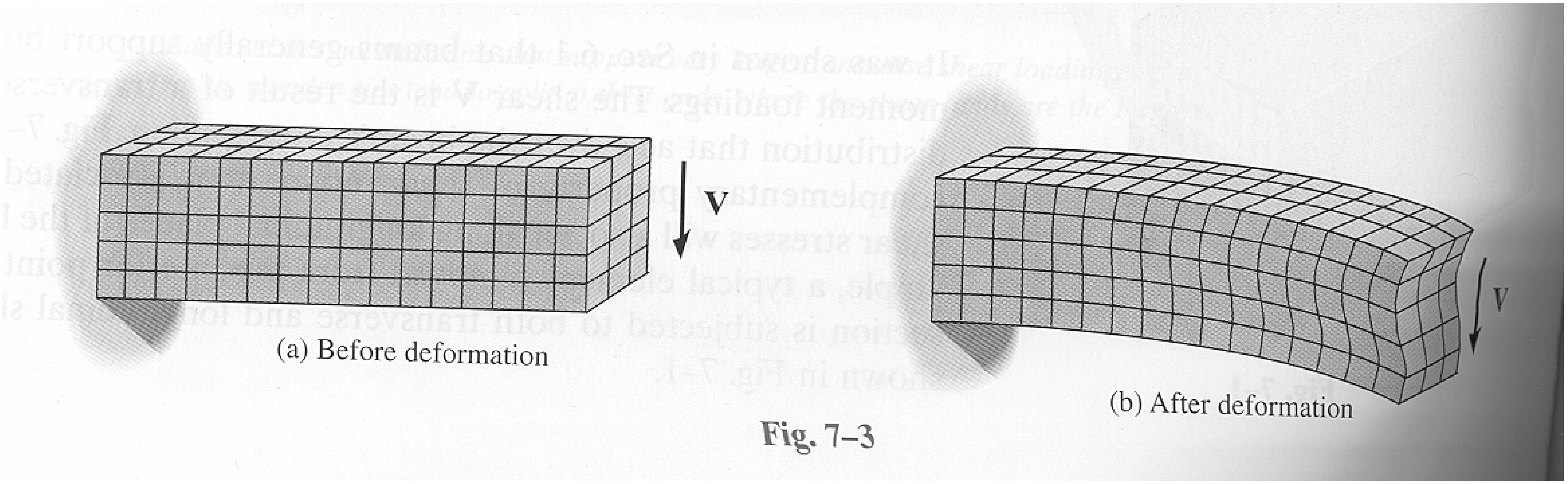
Longitudinal Shear Stress

- The boards can be seen to slide relative to each other at the contact surfaces if they are not bonded.
- Hence when they are bonded a longitudinal shear stress is developed at the contact surface.



Transverse Shear Stress

- Shear stress results in shear strain – hence distortions in the cross section.
- These deformations are non-uniform and tend to warp the section – thus plane sections do not remain plane

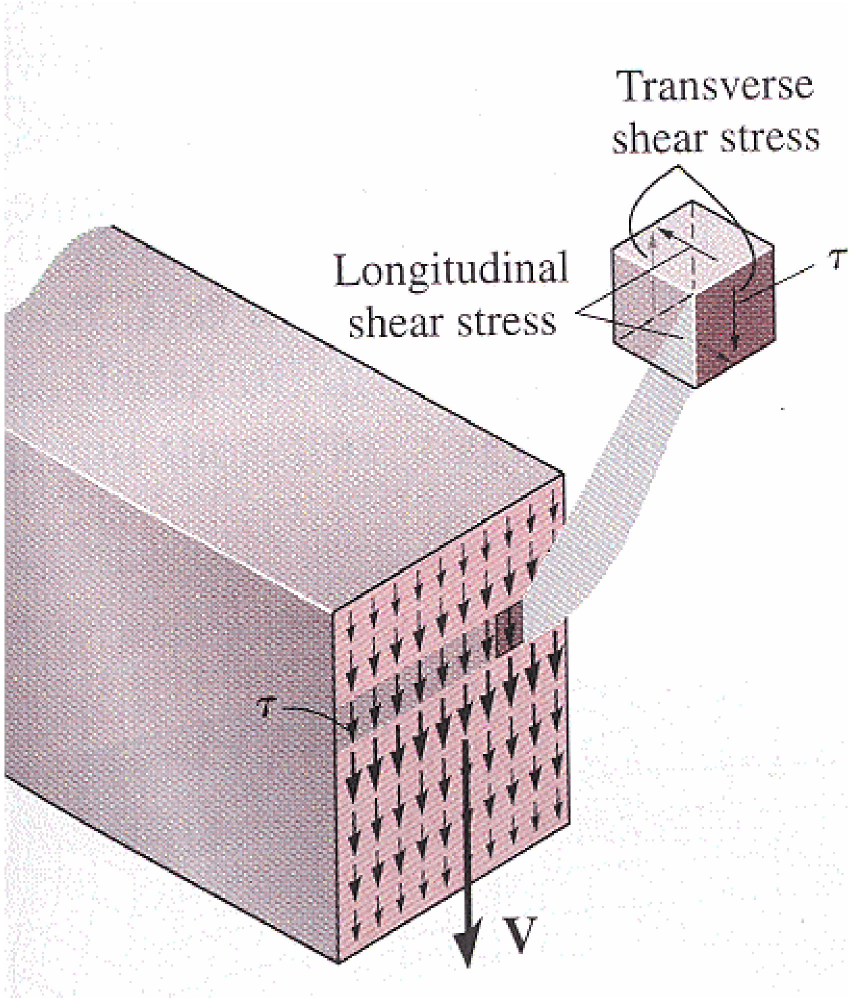


(a) Before deformation

(b) After deformation

Fig. 7–3

Shear Stresses in Beams



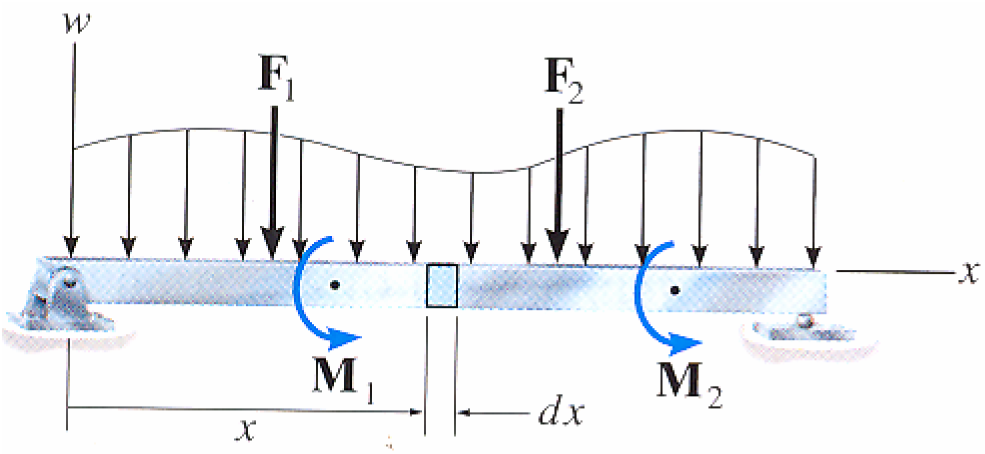
- The vertical (transverse) shear stress on the cross section exists together with the complementary shear stress in the longitudinal direction on the horizontal (longitudinal) sections.
- The transverse and longitudinal shear stress are complementary and numerically equal.

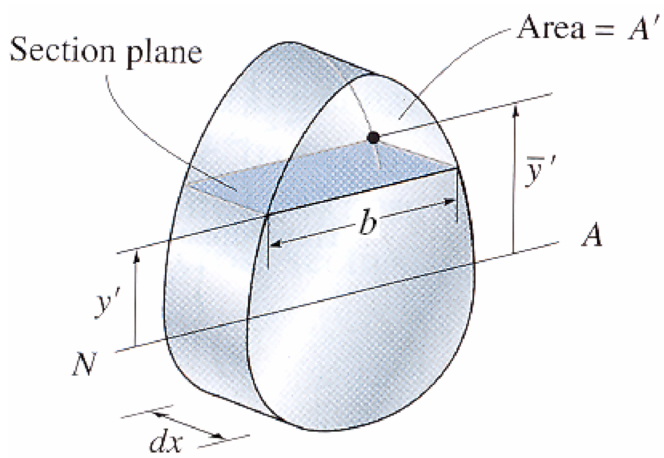
The relationship between V and M

- Consider a general beam under various external forces, the relationship between shear force and bending moment can be expressed by

$$V = \frac{dM}{dx}$$

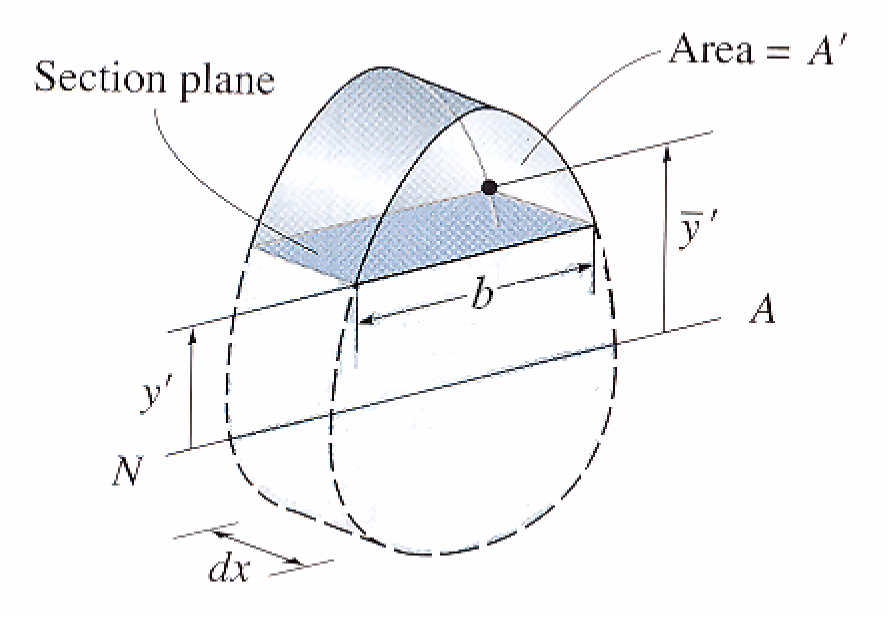
- To determine the shear stress distribution over the cross-section of the beam, we study a portion of the element taken from the beam.





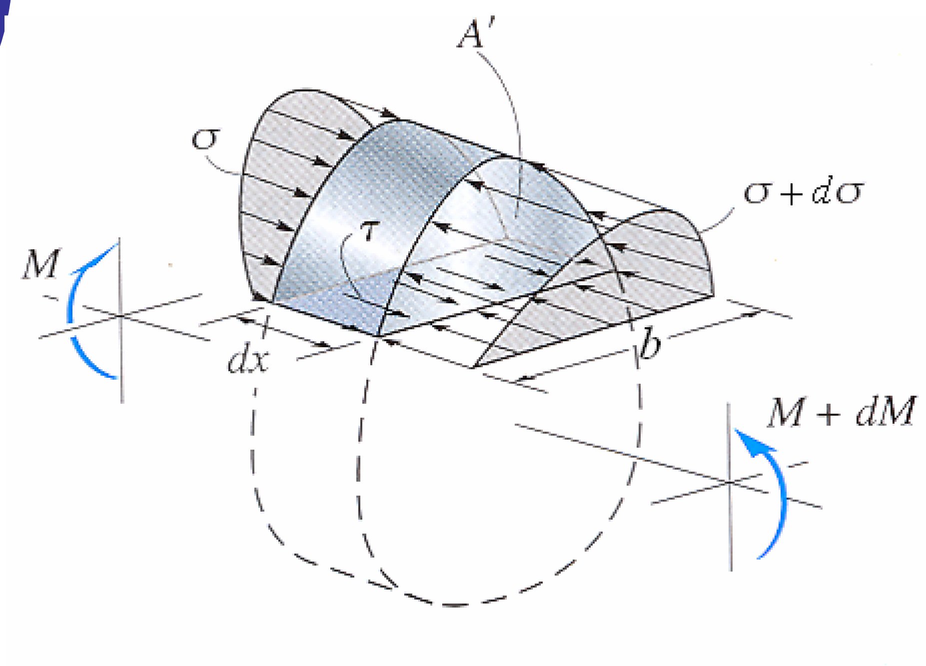
Geometry of the top segment

- Consider the top segment of the element.
- The length of the cut is ***b*** and the area is ***A***'.
- ***y'*** is the distance to the cut measured from the neutral axis.
- $\bar{y}'$ is the distance from the neutral axis to the centroid of *A*'.

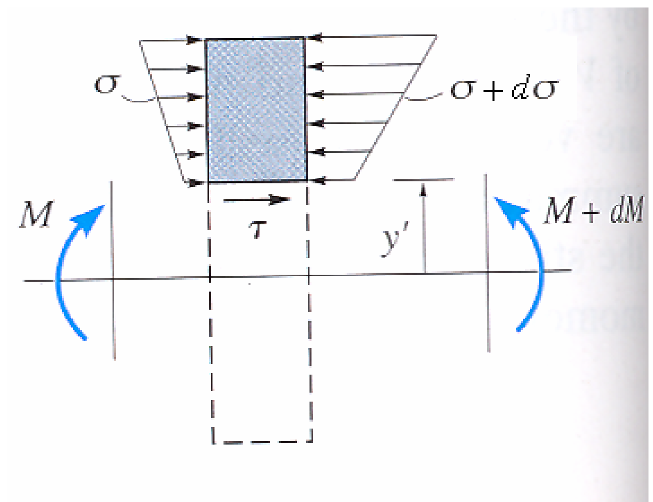


Free-body diagram at horizontal direction

- At both cross-sectional sides, there are bending stresses σ and $\sigma + d\sigma$
- The horizontal shear stress τ acts over the bottom face.



Three-dimensional view

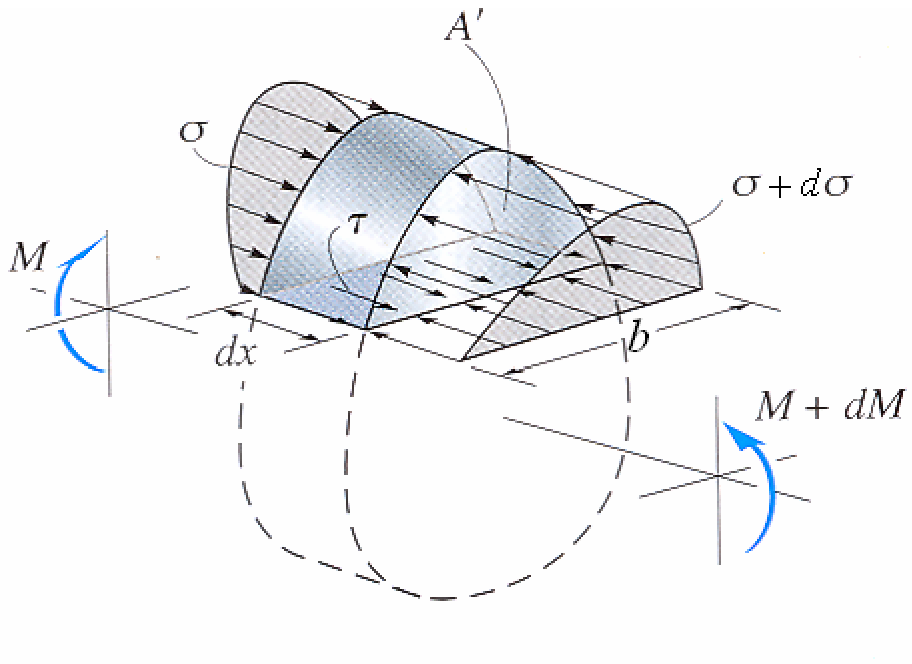


Profile view

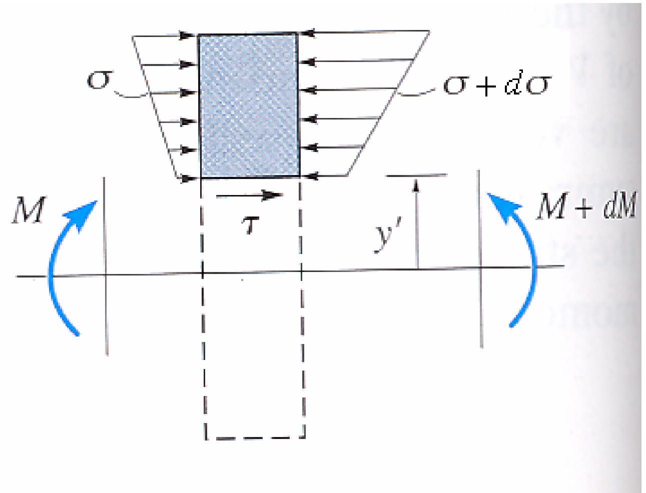
Horizontal force equilibrium

$$\xrightarrow{+} \sum F_x = 0: \quad \int_{A'} \sigma dA + \tau(bdx) - \int_{A'} (\sigma + d\sigma) dA = 0$$

$$\tau(bdx) = \int_{A'} d\sigma dA$$



Three-dimensional view



Profile view

Shear stress due to shear force

$$\tau(bdx) = \int_{A'} d\sigma dA$$

Substituting the compression stress $\sigma = \frac{My}{I}$ into the above equation,

$$\tau(bdx) = \left(\frac{dM}{I}\right)\int_{A'} ydA \quad \text{or} \quad \tau = \frac{1}{Ib}\left(\frac{dM}{dx}\right)\int_{A'} ydA$$

The bending moment is related with the shear force as $V = \frac{dM}{dx}$

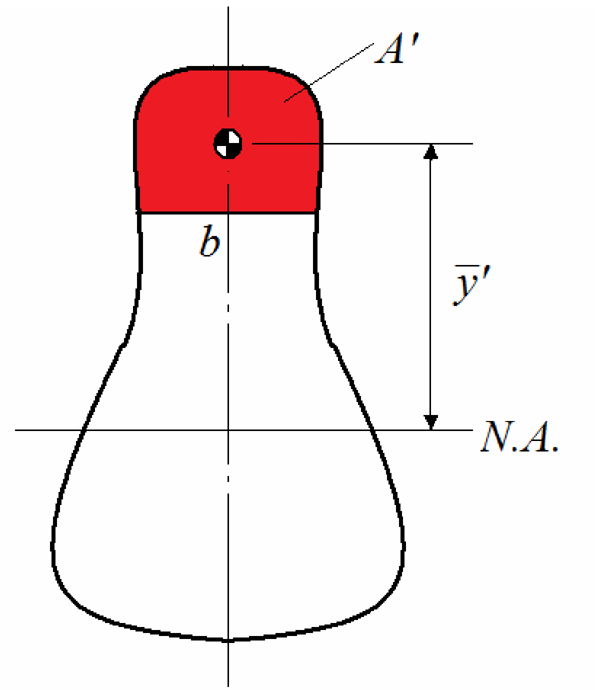
$$\tau = \frac{V}{Ib}\int_{A'} ydA$$

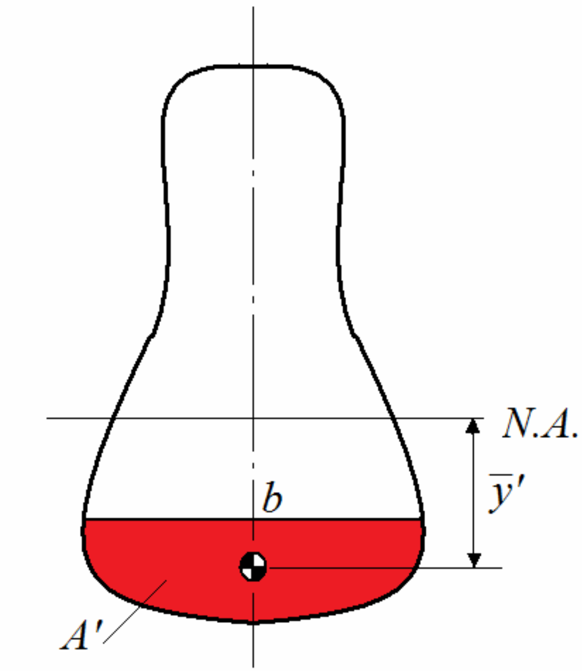
Integration term

$$\tau = \frac{V}{Ib}\int_{A'} y\,dA$$

The term can be simplified by

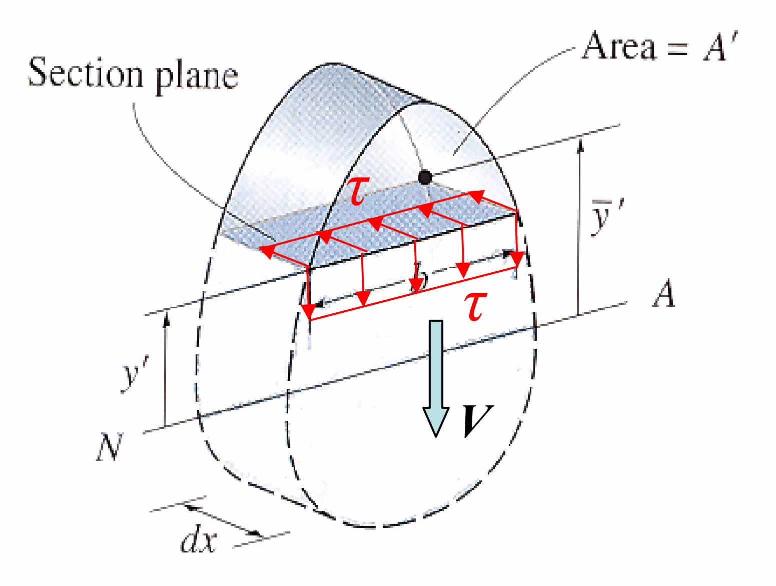
$$\int_{A'} y\,dA = \bar{y}'A'$$





Shear stress formula

$$\tau = \frac{VA'\overline{y}'}{Ib}$$



The expression evaluates the shear stress on the plane formed by an imaginary cut of width b.

V – the vertical shear force

A' – is the partial cross-section area.

$\overline{y}'$ – is the distance to the centroid of the partial cross-section measured from N.A.

I – Moment of interia

b – the width of the cut.

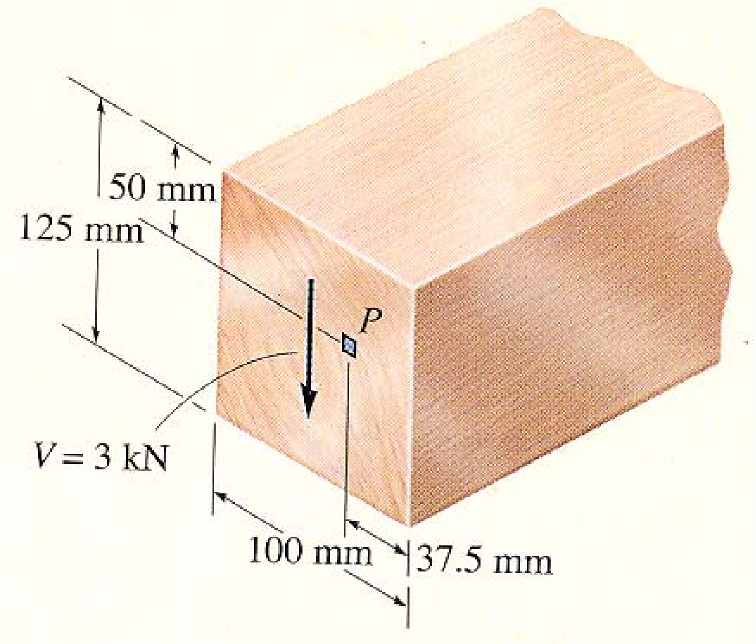
Example 1

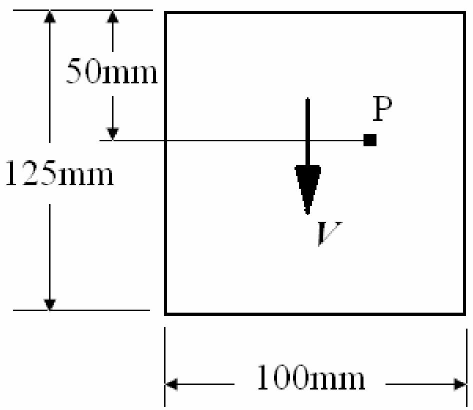
The beam is subjected to a resultant internal vertical shear force of V=3kN.

(a) Determine the shear stress in the beam at point *P*.

(b) Compute the maximum shear stress in the beam.

$$\tau = \frac{VA'\bar{y}'}{Ib}$$



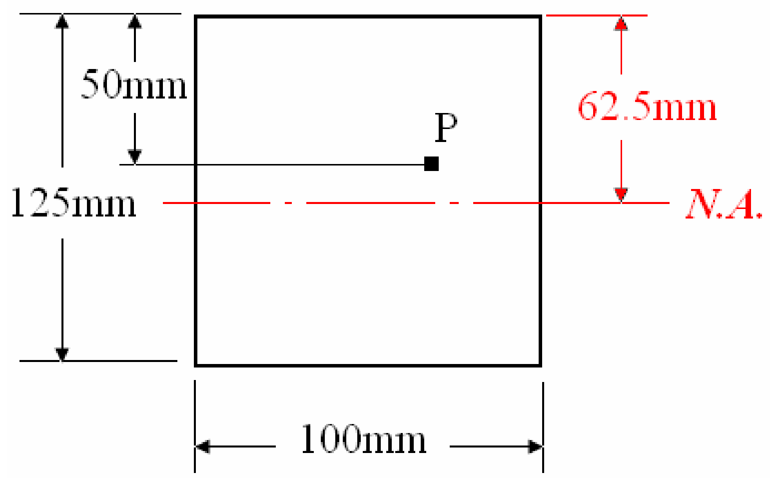


Example 1-Section Properties

(1) Neutral axis and moment of interia

The neutral axis of the entire cross-section is 62.5mm from the top. The moment of inertia about the neutral axis is

$$I=\frac{bh^3}{12}=\frac{100\times125^3}{12}=16.28\left(10^6\right)\text{mm}^4$$

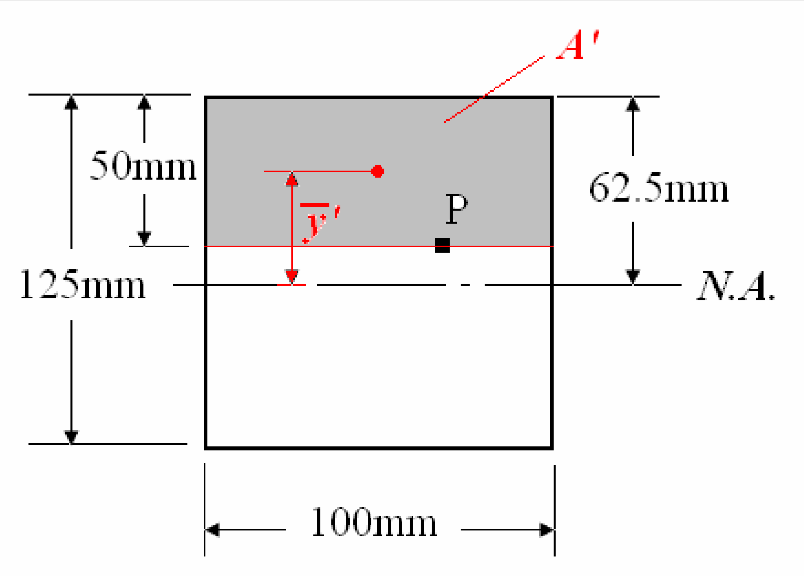


Example 1- Section Properties

(1) Neutral axis and moment of interia

The neutral axis of the entire cross-section is 62.5mm from the top. The moment of inertia about the neutral axis is

$$I = \frac{bh^3}{12} = \frac{100 \times 125^3}{12} = 16.28\left(10^6\right)\text{mm}^4$$



(2) A' and $\bar{y}'$

Draw a horizontal section line through point P, the partial section area A' is shown in the shaded area

$$A' = 100 \times 50 = 5000\text{mm}^2$$

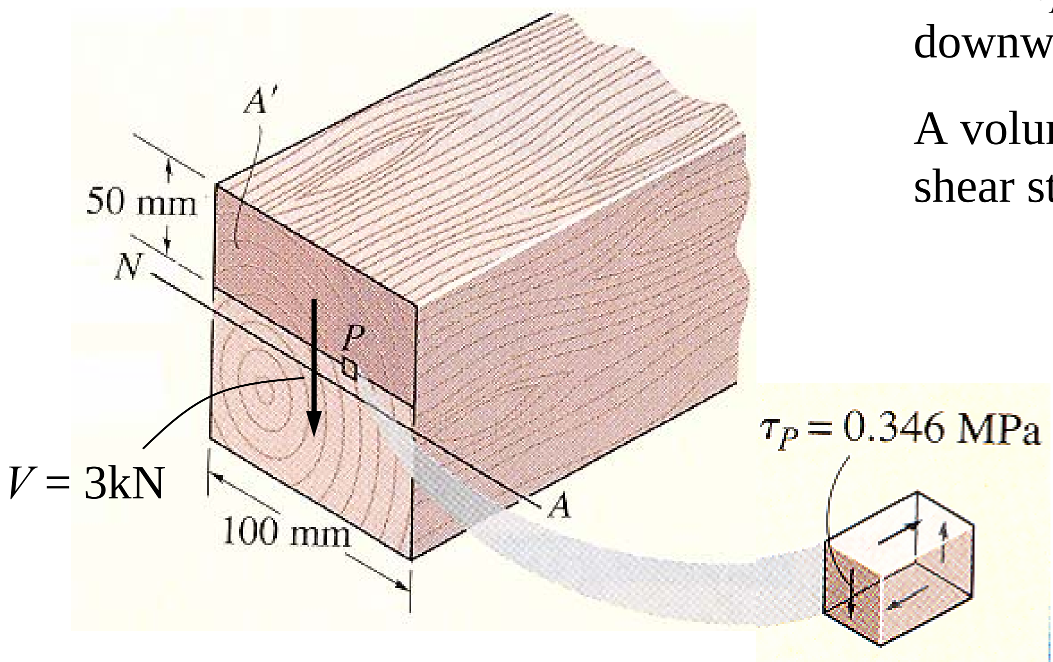
$\bar{y}'$ is defined the distance from the centre of the partial section to $N.A.$

$$\bar{y}' = 25 + (62.5 - 50) = 37.5\text{mm}$$

Example 1 - Shear Stress

(3) The shear force at the section is V=3kN.
Applying the shear formula,

$$\tau_P = \frac{VA'\bar{y}'}{Ib} = \frac{3000 \times 5000 \times 37.5}{16.28(10^6) \times 100} = 0.346\text{N/mm}^2 = 0.346\text{MPa}$$



Since τ_P contributes to V, it acts downward at P on the cross section.

A volume element at P would have shear stresses acting on as shown.

Example 1 - Maximum shear stress

$$\tau = \frac{VA'\bar{y}'}{Ib}$$

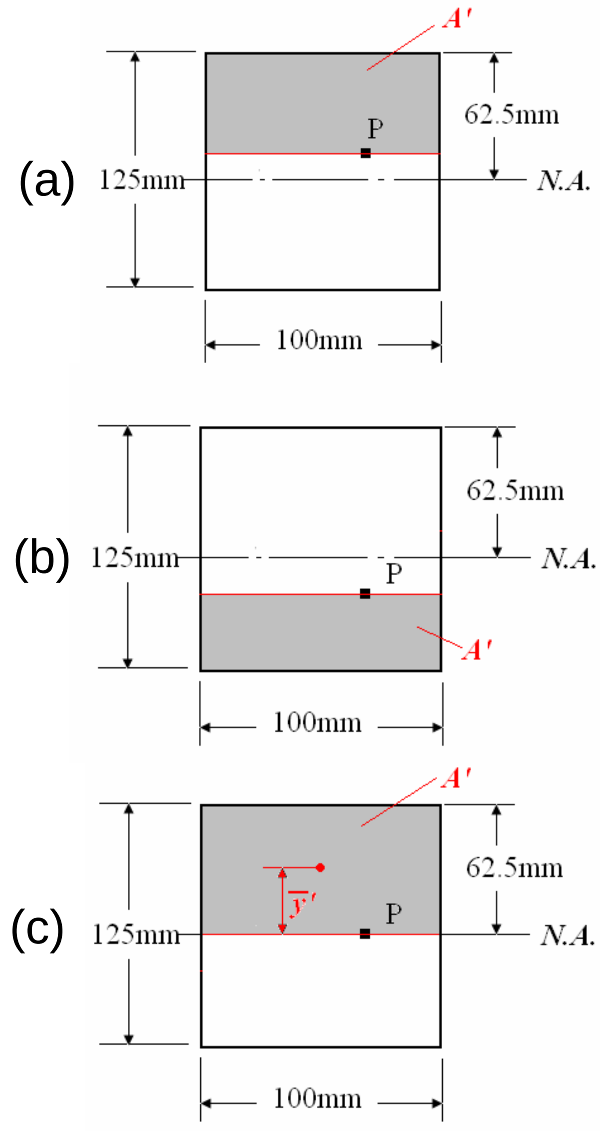
The shear stress increases with the partial area A'. When the interested point is above or below N.A., the partial area A' is shown in (a) and (b) respectively.

So, the maximum shear stress occurs at the neutral axis, since A' is largest as shown in (c). In this case

$$A' = 100 \times 62.5 = 6250\text{mm}^2$$

$$\bar{y}' = 62.5/2 = 31.25\text{mm}$$

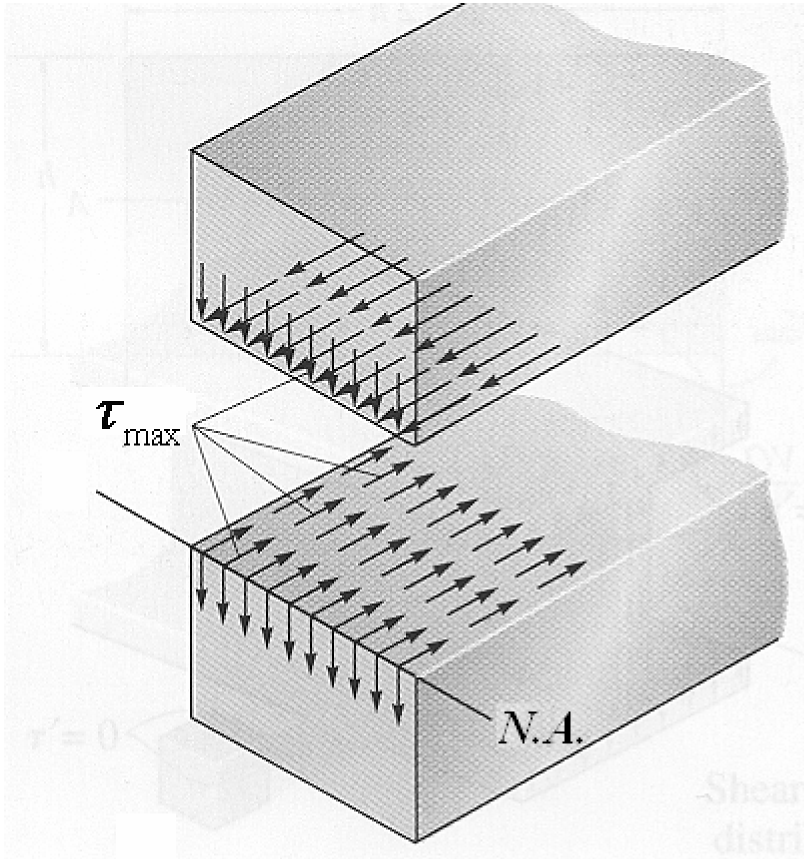
$$\tau_{\max} = \frac{3000 \times 6250 \times 31.25}{16.28(10^6) \times 100} = 0.360\text{MPa}$$



Maximum shear stress

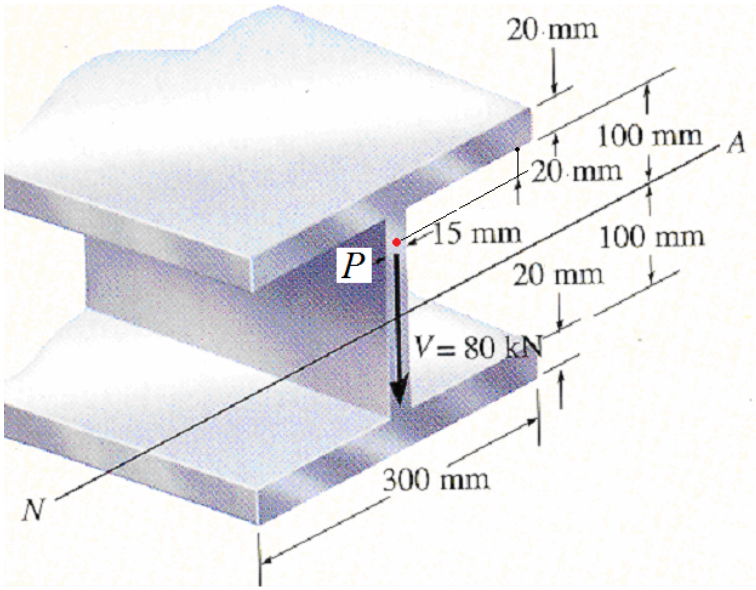
$$\tau_{\max} = 0.360\text{MPa}$$

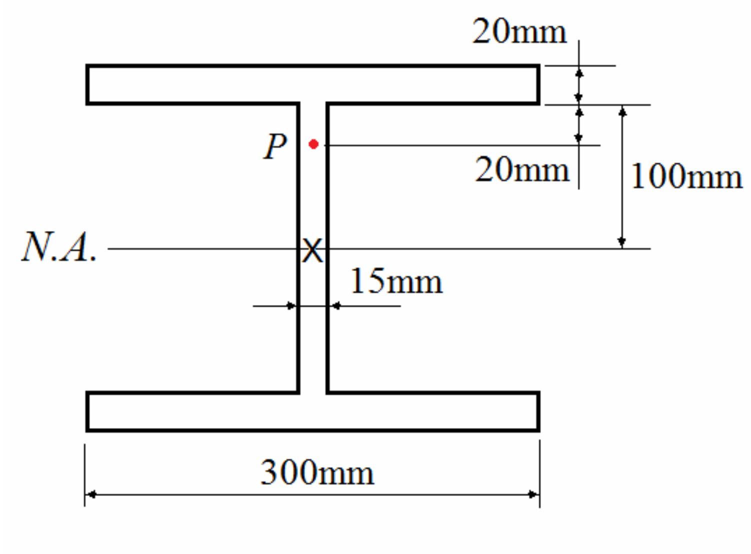
If we imagine splitting the beam by a horizontal plane through its neutral axis, the complementary shear stresses (horizontal shear stress) are apparent.



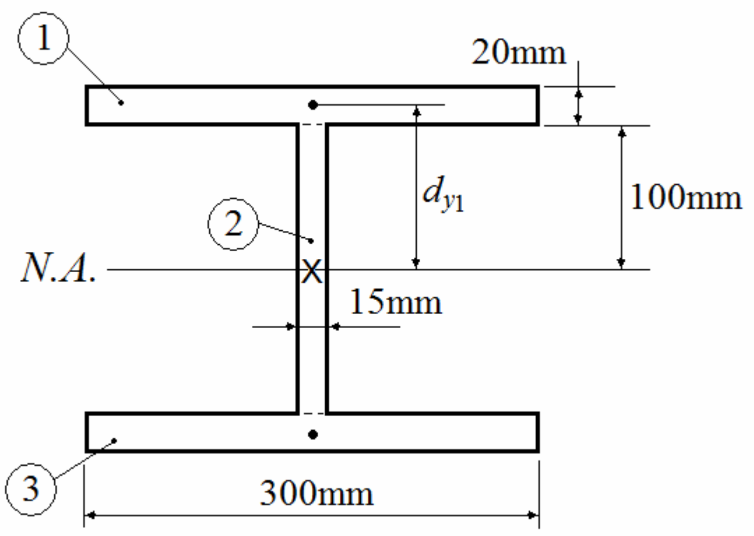
Example 2

A steel I-beam has the dimensions shown in the figure and is subjected to a shear force V=80kN. (a) determine the shear stress at point P; (b) determine the shear force resisted by the top flange.





Example 2 – moment of inertia



$I = 2\times(200000+72600000)+10000000$

$= 155600000(mm^4)$

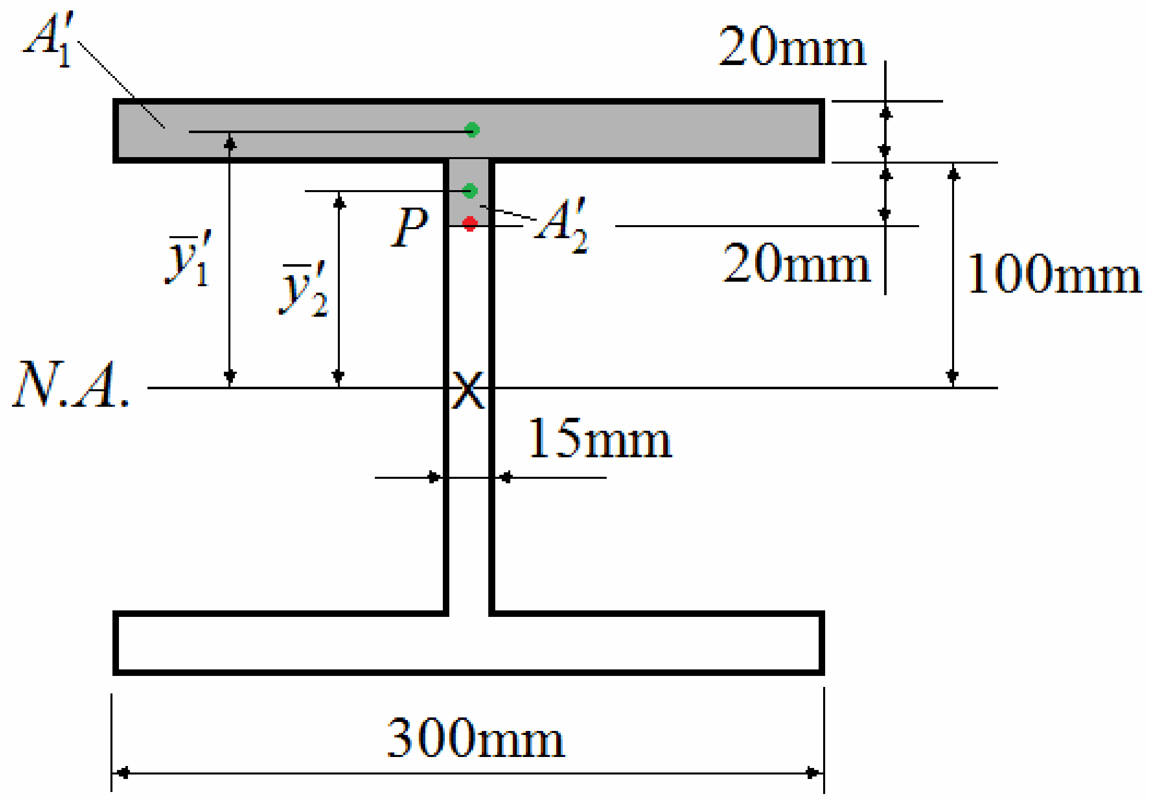
Seg. No.	I_{cxx} (mm^4)	A_i (mm^2)	d_{yi} (mm)	$A_i d_{yi}^2$ (mm^4)
1	$1/12 \times 300 \times 20^3 = 200000$	$300\times20 = 6000$	$100+10= 110$	72600000
2	$1/12 \times 15\times200^3 = 10000000$	$15 \times 200 = 3000$	0	0
3	$1/12 \times 300 \times 20^3 = 200000$	$300\times20 = 6000$	$100+10= 110$	72600000

Example 2(a) – A' and $\bar{y}'$

$$A'\bar{y}' = \sum A_i'\bar{y}_i' = A_1'\bar{y}_1' + A_2'\bar{y}_2'$$

$$= (300\times 20)\times(100+10) + (15\times 20)\times(100-10)$$

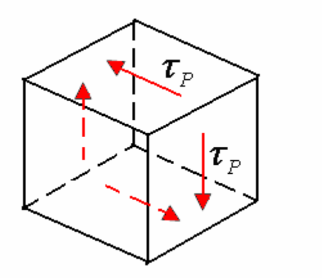
$$= 660000 + 27000 = 687000\text{mm}^3$$

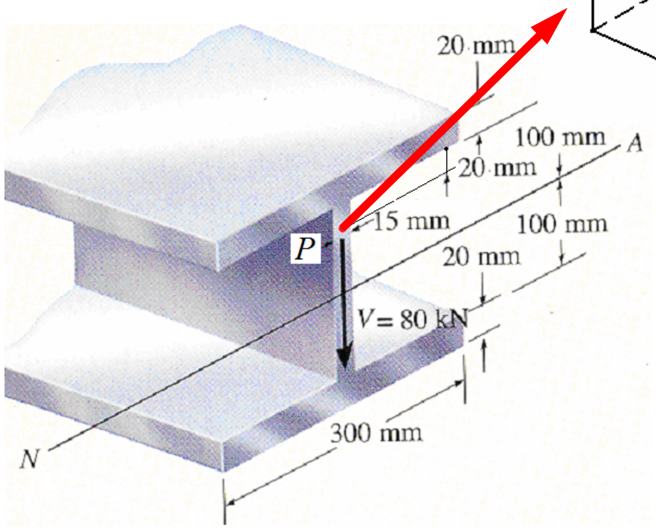


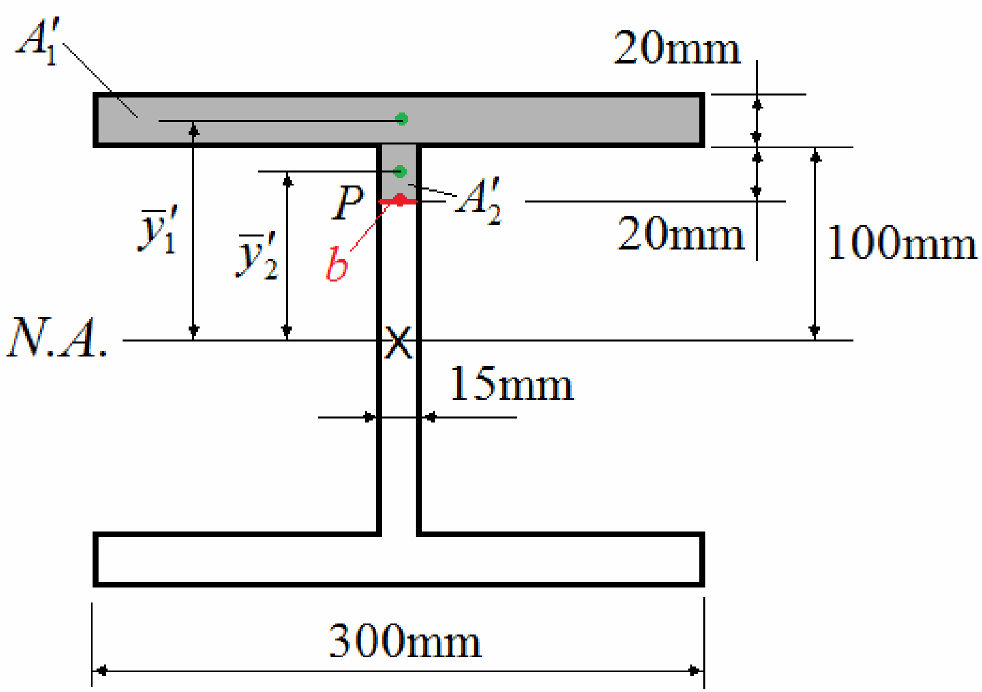
Example 2(a) – Shear stress

$$\tau_P = \frac{VA'\bar{y}'}{Ib} = \frac{80000 \times 687000}{155600000 \times 15} = 23.55(\text{N/mm}^2) = 23.55\text{MPa}$$

A volume element at *P*:





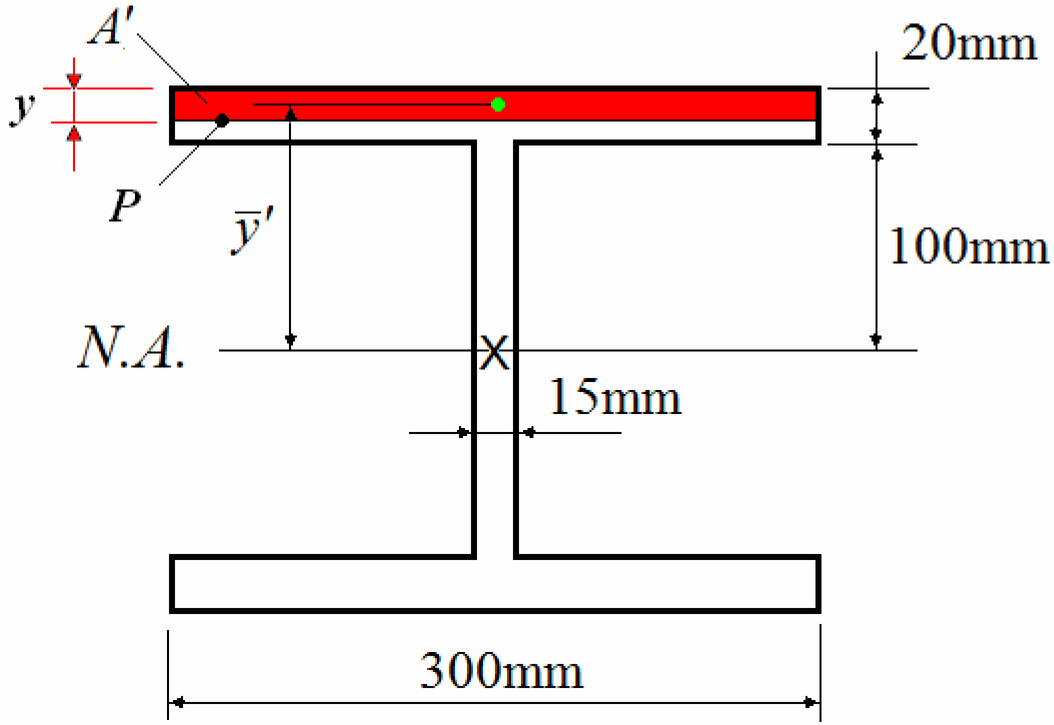


Example 2(b) – shear force at top flange

To determine the shear force caused by the top flange, we formulate the shear stress at the arbitrary location y within the top flange.

$$A' = 300y$$

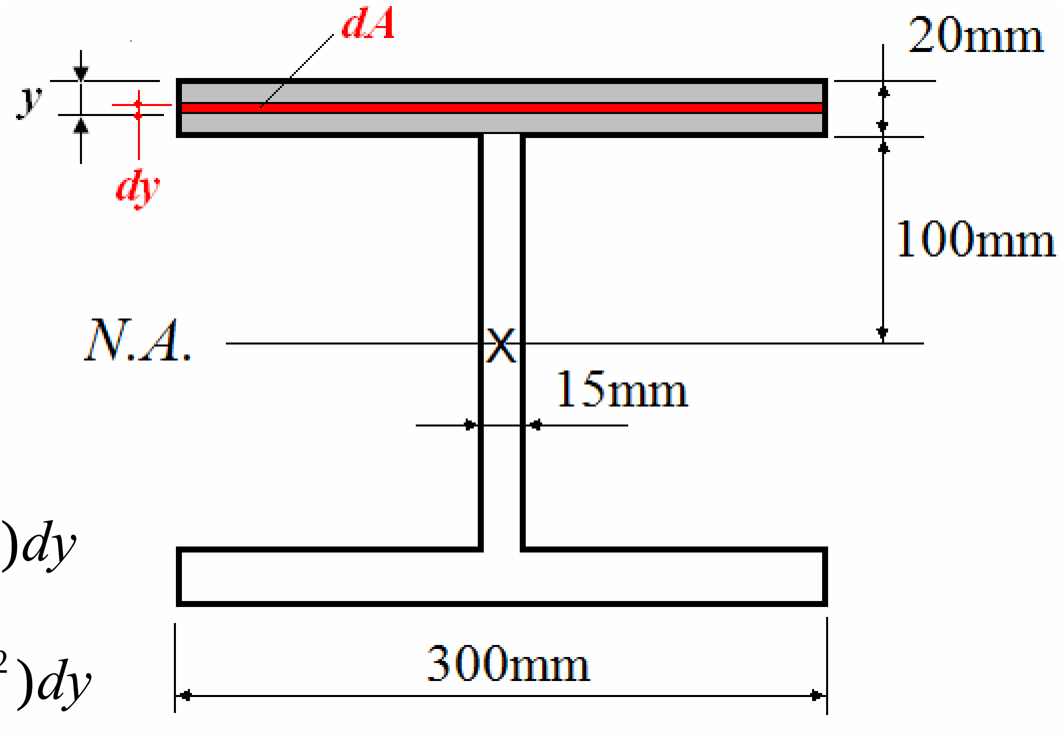
$$\bar{y}' = 120 - y/2$$



$$\tau(y) = \frac{VA'\bar{y}'}{Ib} = \frac{80000 \times 300y \times (120 - y/2)}{155600000 \times 300}$$

Example 2(b) – shear force at top flange

$$\tau(y) = 0.062y - 0.0003y^2$$



This shear stress acts on the area strip dA=300dy. Therefore the shear force resisted by the top flange is

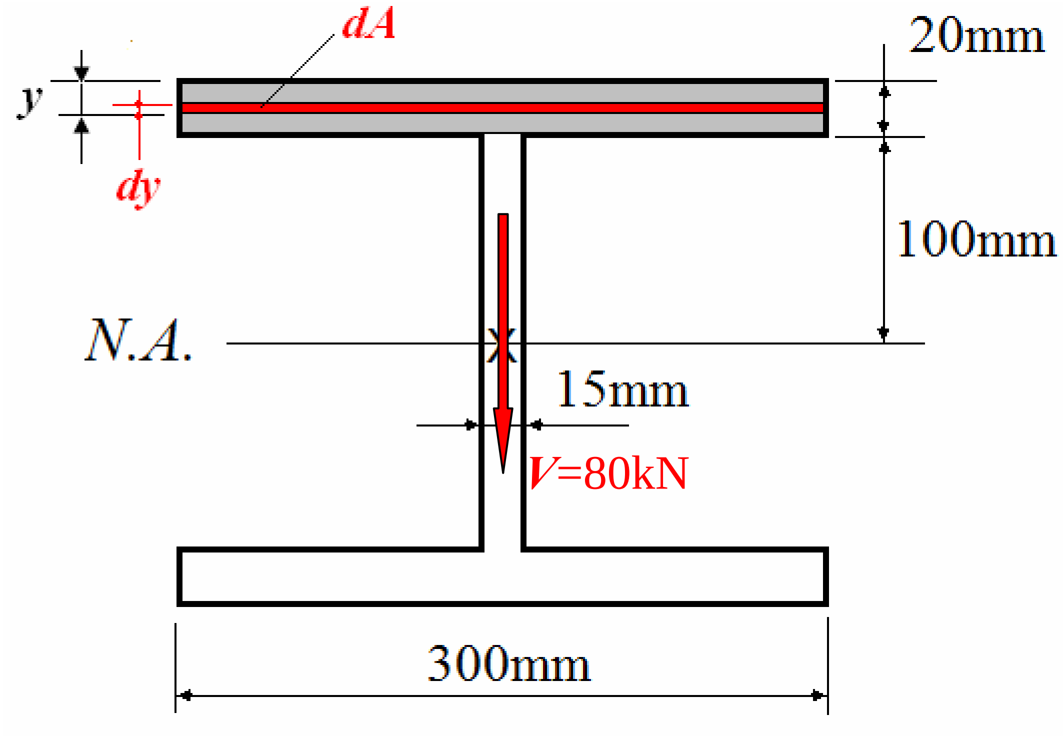
$$V_{TF} = \int_A \tau(y)dA = 300\int_A \tau(y)dy$$

$$= 300\int_0^{20} (0.062y - 0.0003y^2)dy$$

$$= 300\left[\frac{0.062}{2}y^2 - \frac{0.0003}{3}y^3\right]_0^{20} = 3480\text{N} \approx 3.5\text{kN}$$

Example 2(b) – shear force at top flange

$$\tau(y) = 0.062y - 0.0003y^2$$



This shear stress acts on the area strip dA=300dy. Therefore the shear force resisted by the top flange is

$$V_{TF} = 3.5\text{kN}$$

How is the total shear force V=80kN allocated?

Top flange: 3.5kN

Bottom flange: 3.5kN

Web: 80-3.5-3.5=73.0kN

Procedure for analysis

- Determine the shear force V (sometime it is given).
- Determine the location of the neutral axis and moment of inertia (I).
- Draw an imaginary horizontal line through the interested point. Measure the width of the cut (b).
- Determine the partial section area (A') above or below the line and $\bar{y}'$ which is the distance to the centroid of A', measured from the neutral axis.
- Using a consistent set of units, calculate the shear stress τ.

$$\tau = \frac{VA'\bar{y}'}{Ib}$$

Units 1: V(N), A' (mm^2), $\bar{y}'$(mm), I(mm^4), b(mm)
the resultant τ (N/mm^2 or MPa)

Units 2: V(N), A' (m^2), $\bar{y}'$(m), I(m^4), b(m)
the resultant τ (N/m^2 or Pa)

……

Maximum shear stress in beams

- For the middle beam, find the location and magnitude of the maximum shear forces from Shear Force Diagrams (all of three cases).
- Calculate the maximum shear stress at the neutral axis using

$$\tau = \frac{VA'\overline{y}'}{Ib}$$

- Comparing the maximum shear stress with the yield stress (350MPa):

  If

  $$\tau_{\max} < \sigma_Y = 350\text{MPa}$$

  OK!!!

- Following the same procedure for the Side Beam.

This is the **End** of your Calculation

Summary of your calculations

- Design loads
- Load combinations (three load cases for purlin)
- S.F.Ds and B.M.Ds for Middle Beam
- S.F.Ds and B.M.Ds for Side Beam

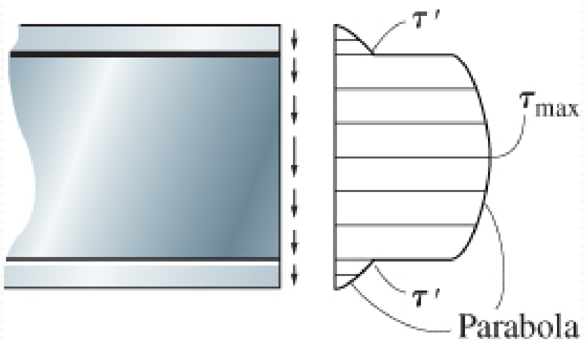
→ Refers to the note of Design Loads

- Maximum bending stresses for Middle Beam and Side Beam → Refers to CIVE1188-T6
- Maximum deflections for Middle Beam and Side Beam (using different load combination) → Refers to CIVE1188-Lect8
- Maximum shear stresses for Middle Beam and Side Beam → Refers to CIVE1188-Lect9

Exam Discussion

Wide-flange beam

- The shear-stress distribution varies parabolically over beam’s depth
- Note there is a **jump** in shear stress at the flange-web junction since x-sectional thickness changes at this pt
- The web carries significantly more shear force than the flanges



Intensity of shear–stress distribution (profile view)

(c)

1. MARKING SCHEME – EXAM

Task – Satisfactory completion of an examination as set out by the University

PART A: ROLES

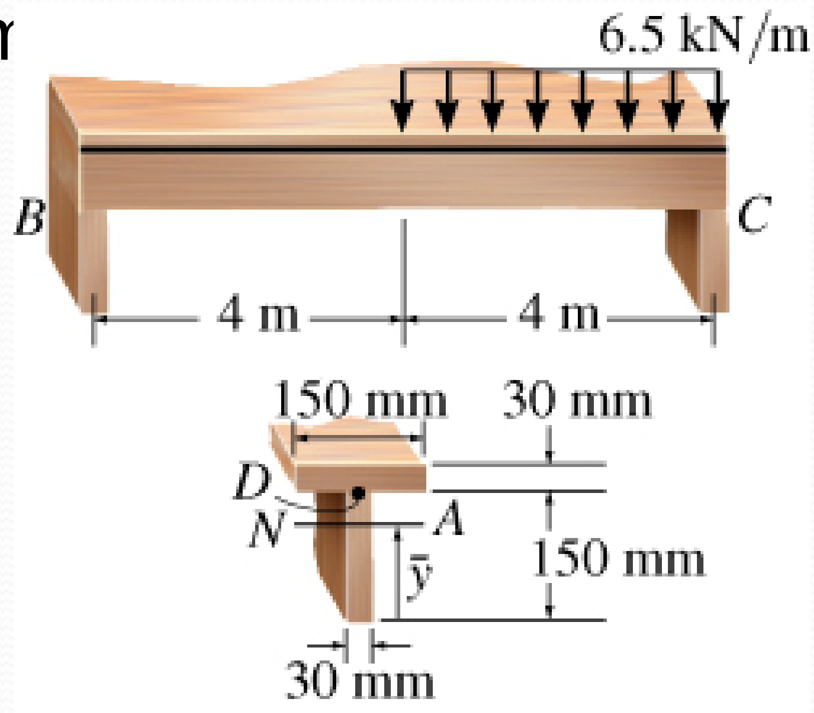
<table>
<tr><th>Student Role</th><th>Lecturer Role</th><th>Remarks</th></tr>
<tr>
<td>1. Adequately prepares and attends the examination.
2. Attempts and answers questions as per the instructions and to the best of their efforts.
3. Completes the examination in a professional manner and as set out by the university.
4. Use only school approved calculators with approved stickers on them.</td>
<td>• Sri will run an exam revision in week 12 and a model solution posted on the blackboard.
• Sri will set the exam and prepare a marking criteria.
• A qualified team may be assigned for marking.
• In case of marking disputes, Sri will verify student claims and advise of the outcome.</td>
<td>• It is the responsibility of students to undertake the exam.
• Exam format - you will need to attempt 5 out of 6 questions set out from ANY of the topics covered in the lecture.
• Questions will be at or above the level of tutorial problems or from the reference book.
• Some conceptual questions similar to Lecture Quiz may also be included.</td>
</tr>
</table>

PART B: MARKING (BASED on marking criteria developed by the lecturer/ marking team, e.g. 50% for methods and 50% for calculations)

50 marks (1)	Between and including 40 marks and 49 marks	Between and including 30 marks and 39 marks	Between and including 20 and 29 marks	Between and including 10 mark and 19 marks	Between and including 0 mark and 9 marks
• Completion of all problems using the methods noted in the marking criteria and a 100% matching of the calculation results	• Completion of at least 4 out of 5 problems as set out in Column 1 or a few small mistakes that are minor (e.g. improper use of units)	• Completion of at least 3 out of 5 problems as set out in Column 1 or 1 or 2 major errors in the calculation and / or concepts is evident, evidence of minor mistakes	• Completion of at least 2 out of 5 problems as set out in Column 1 or 1 or 2 major errors in calculation and / or concepts is evident and minor errors are evident	• Completion of at least 1 out of 5 problems as set out in Column 1 or 2 or 3 major errors in calculation and / or concepts is evident and many minor errors are evident	• No problems complete d as noted in Column 1 or too many errors and incomplet ions, no attempts etc.,
Lecturer / marker reserves the right to exercise academic / expert judgement in all of the above grading (e.g. what constitutes as major or minor error). Consequential marks are solely at the discretion of the lecturer / marker. To obtain consequential marking full problem must be completed. In case of disputes with exam marking, RMIT procedures and policies as advised the Head of Discipline or the Dean(s) will be resorted for further advice.					

Example 7.3

Beam shown is made from two boards. Determine the maximum shear stress in the glue necessary to hold the boards together along the seams where they are joined. Supports at B and C exert only vertical reactions on the beam.

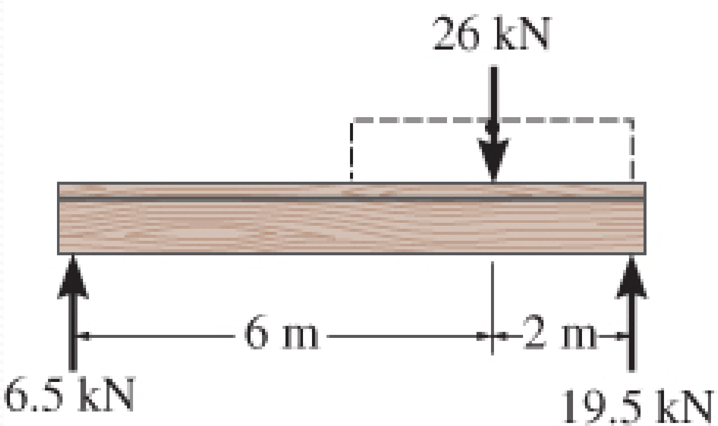


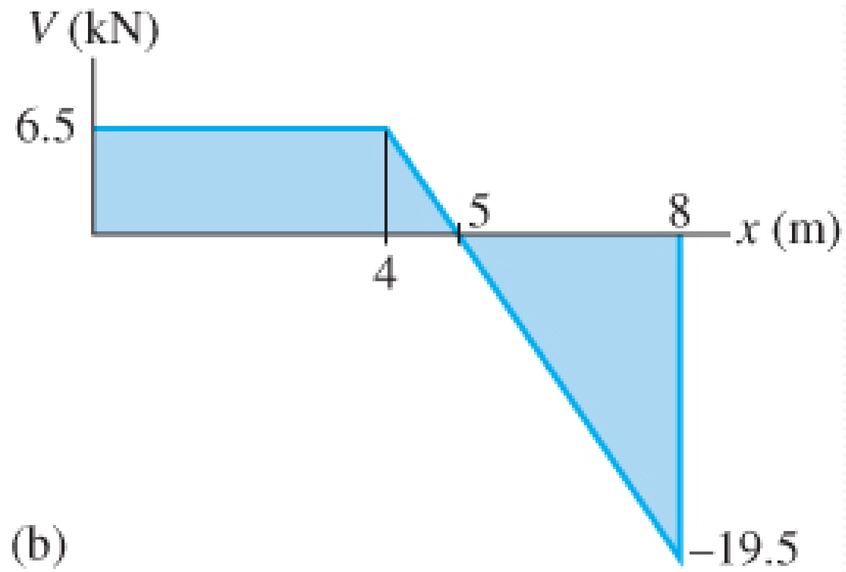
(a)

EXAMPLE 7.3 (SOLN)

Internal shear

Support reactions and shear diagram for beam are shown below. Maximum shear in the beam is 19.5 kN.





(b)

Section properties

The centroid and therefore the neutral axis will be determined from the reference axis placed at bottom of the x-sectional area. Working in units of meters, we have

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \ldots = 0.120 \text{ m}$$

Thus, the moment of inertia, computed about the neutral axis is,

$$I = \ldots = 27.0(10^{-6}) \text{ m}^4$$

EXAMPLE 7.3 (SOLN)

Section properties

The top board (flange) is being held onto the bottom board (web) by the glue, which is applied over the thickness $t = 0.03\text{m}$. Consequently A' is defined as the area of the top board, we have

$$Q = \bar{y}'A' = [(0.180\text{ m} - 0.015\text{ m} - 0.120\text{ m}](0.03\text{ m})(0.150\text{ m})$$

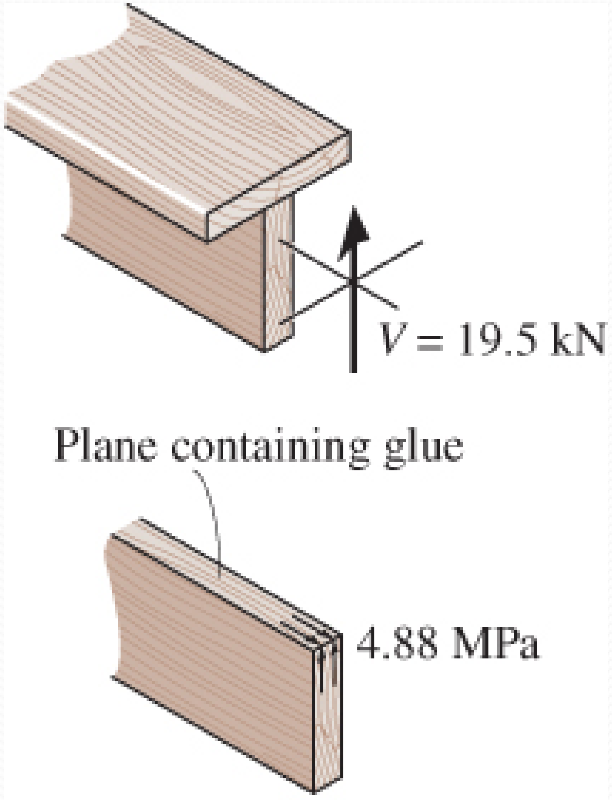
$$Q = 0.2025(10^{-3})\text{ m}^3$$

EXAMPLE 7.3 (SOLN)

Shear stress

Using above data, and applying shear formula yields

$$\tau_{max} = \frac{VQ}{It} = \ldots = 4.88 \text{ MPa}$$



(c)

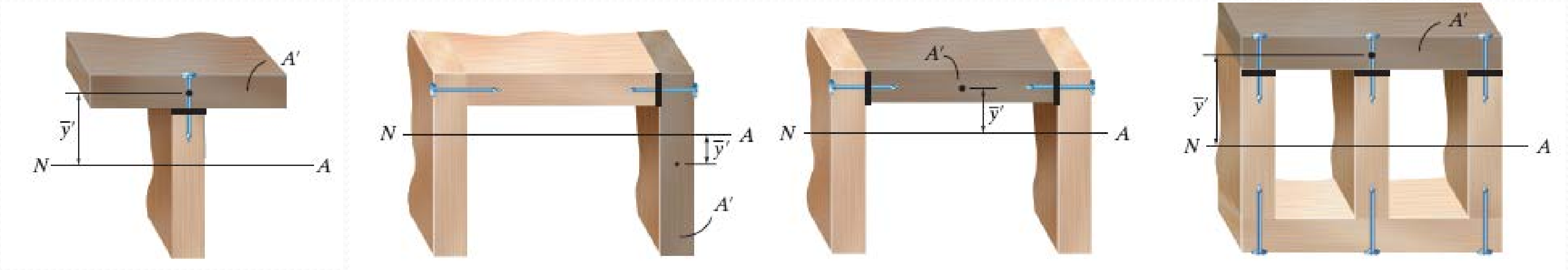
Shear stress acting at top of bottom board is shown here.
It is the glue's resistance to this lateral or *horizontal shear stress* that is necessary to hold the boards from slipping at support C.

Shear Flow in Built-Up Members

- For fasteners it is necessary to know the shear force by the fastener along the member's *length*.
- This loading is referred as the ***shear flow q***, measured as a force per unit length.

$$q = \frac{VQ}{I}$$

q = shear flow
V = internal resultant shear
I = moment of inertia of the *entire* cross-sectional area



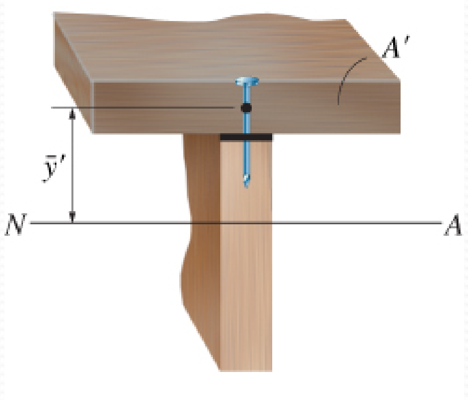
Shear Flow in Built-Up Members

IMPORTANT

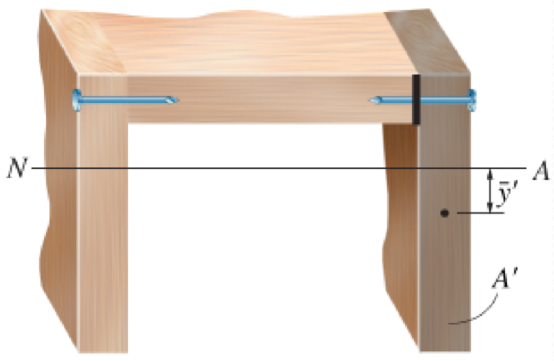
- **Shear flow is a measure of force per unit length along a longitudinal axis of a beam.**
- **This value is found from the shear formula and is used to determine the shear force developed in fasteners and glue that holds the various segments of a beam together**

Shear Flow in Built-Up Members

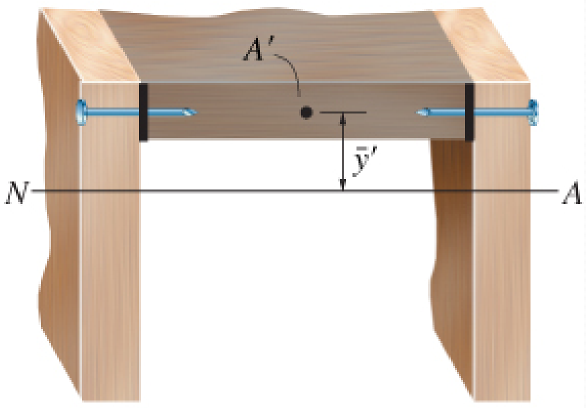
- Note that the fasteners in (a) and (b) supports the calculated value of q
- And in (c) each fastener supports $q/2$
- In (d) each fastener supports $q/3$



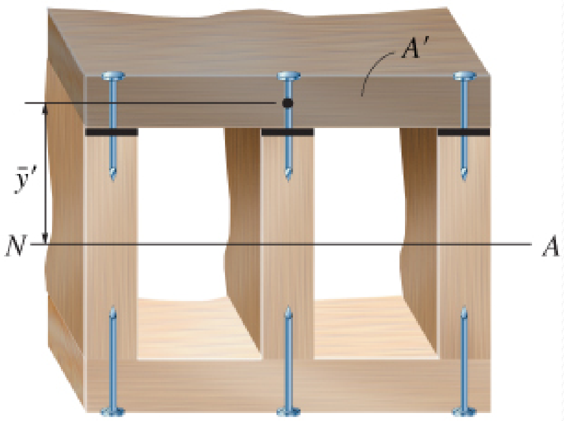
(a)



(b)



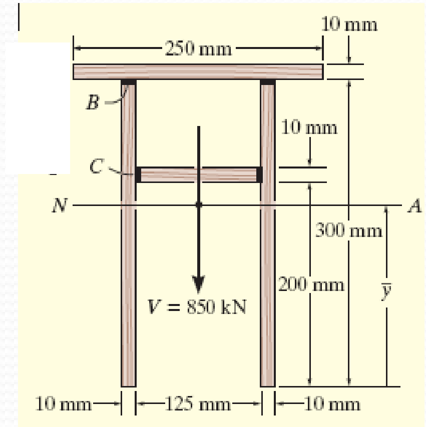
(c)



(d)

Example 7.4

The beam is constructed from four boards glued together. If it is subjected to a shear of V = 850 kN, determine the shear flow at *B* and *C* that must be resisted by the glue.



Solution:

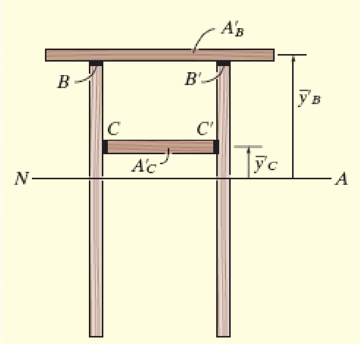
The neutral axis (centroid) will be located from the bottom of the beam,

$$\bar{y} = \frac{\sum \tilde{y} A}{\sum A} = 0.1968 \text{ m}$$

The moment of inertia computed about the neutral axis is thus $I = 87.52\left(10^{-6}\right)\text{m}^4$

Since the glue at *B* and holds the top board to the beam

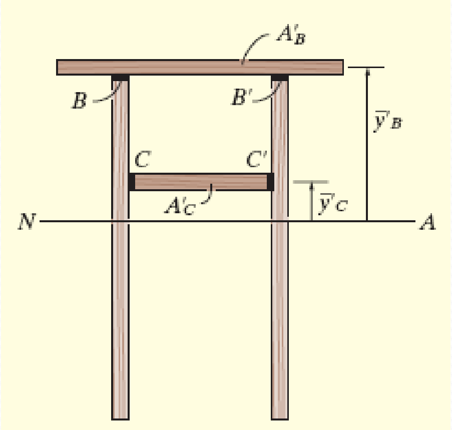
$$Q_B = \bar{y}'_B A'_B = [0.305 - 0.1968](0.250)(0.01) = 0.271\left(10^{-3}\right)\text{m}^3$$



Solution:

Likewise, the glue at *C* and *C*’ holds the inner board to the beam

$$Q_C = \overline{y}'_C\,A'_C = [0.205 - 0.1968](0.125)(0.01) = 0.01026\left(10^{-3}\right)\text{m}^3$$



Therefore the shear flow for BB’ and CC’,

$$q'_B = \frac{VQ_B}{I} = \frac{(850)\left(0.271\times10^{-3}\right)}{87.52\times10^{-6}} = 2.63\,\text{MN/m}$$

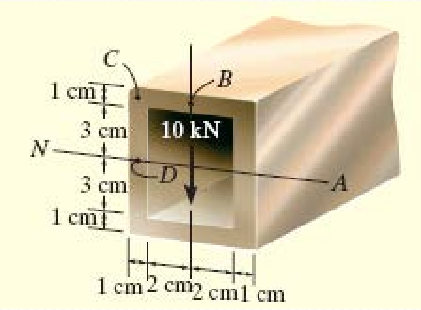
$$q'_C = \frac{VQ_C}{I} = \frac{(850)\left(0.01026\times10^{-3}\right)}{87.52\times10^{-6}} = 0.0996\,\text{MN/m}$$

Since *two seams* are used to secure each board, the glue per meter length of beam at each seam must be strong enough to resist *one-half* of each calculated value of q’.

$$q_B = 1.31\,\text{MN/m} \quad \text{and} \quad q_C = 0.0498\,\text{MN/m} \quad \text{(Ans)}$$

Example 7.7

The thin-walled box beam is subjected to a shear of 10 kN. Determine the variation of the shear flow throughout the cross section.



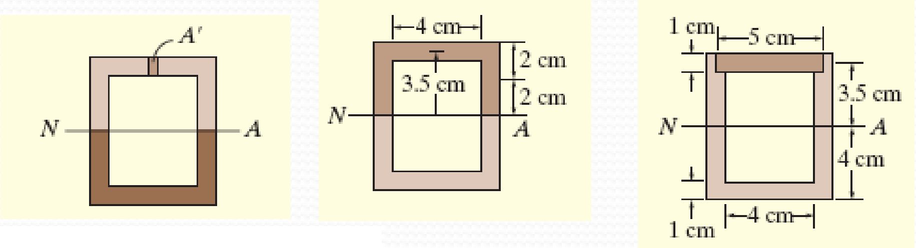
Solution:

The moment of inertia is $I = \frac{1}{12}(6)(8)^3 - \frac{1}{12}(4)(6)^3 = 184 \text{ mm}^4$

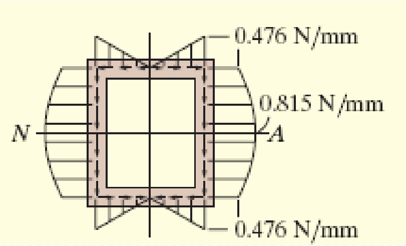
For point *B*, the area $A' \approx 0$ thus $q'_B = 0$.

Also, $Q_C = \bar{y}A' = (3.5)(5)(1) = 17.5 \text{ cm}^3$

$Q_D = \sum \bar{y}A' = 2(2)(1)(4) = 30 \text{ cm}^3$



For point *C*, $q_C = \frac{VQ_C}{I} = \frac{10(17.5/2)}{184} = 0.951 \text{ kN/cm} = 91.5 \text{ N/mm}$



The shear flow at *D* is

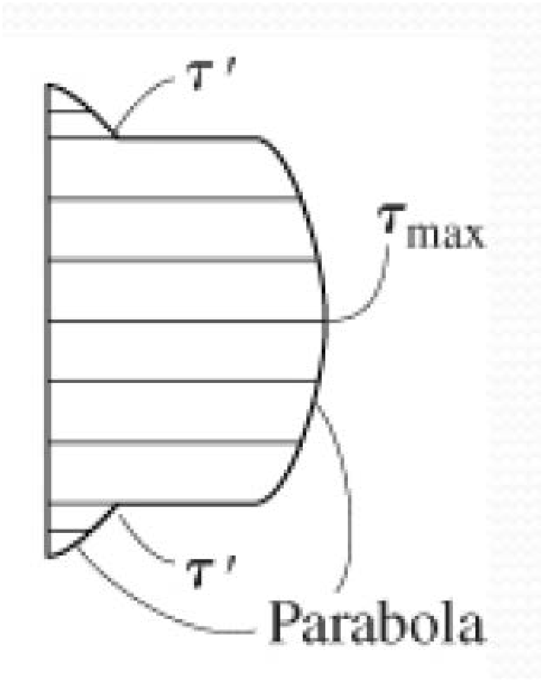
$$q_D = \frac{VQ_D}{I} = \frac{10(30/2)}{184} = 1.63 \text{ kN/cm} = 163 \text{ N/mm}$$

Shear stresses can exist in

a) Only in the longitudinal axis of the beam
b) Only in the transverse direction
c) Both longitudinal and transverse
d) Exists similar to bending stresses

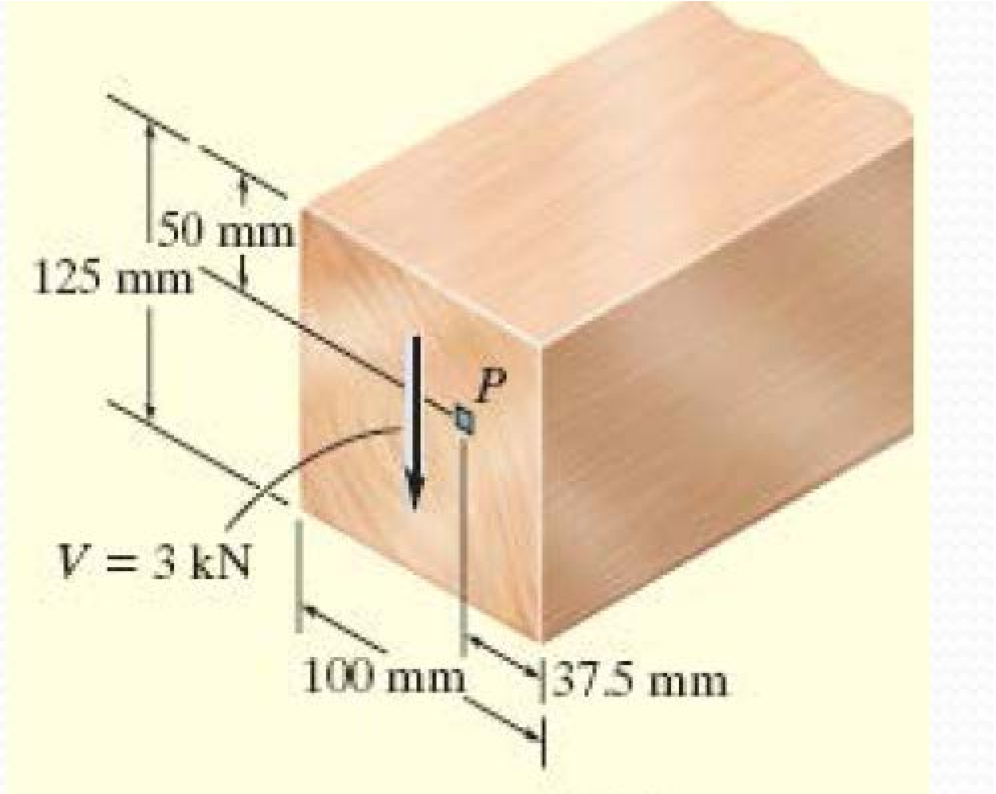
The following shear stress distribution sketch is common for

a) Normal rectangular beams
b) I beams
c) Circular beams
d) Wooden beams



When calculating shear stress for point P, what is the distance “*y*” (i.e. centroid distance)?

a) 50 mm
b) 75 mm
c) 125 mm
d) 37.5 mm



(a)