Chapter 4

Dynamics: Newton’s Laws of Motion
Units of Chapter 4

- Force
- Newton’s First Law of Motion
- Mass
- Newton’s Second Law of Motion
- Newton’s Third Law of Motion
- Weight—the Force of Gravity; and the Normal Force
- Solving Problems with Newton’s Laws: Free-Body Diagrams
- Problem Solving—A General Approach

4-1 Force

A force is a push or pull. An object at rest needs a force to get it moving; a moving object needs a force to change its velocity.
4-1 Force

Force is a vector, having both magnitude and direction.

The magnitude of a force can be measured using a spring scale.

4-2 Newton’s First Law of Motion

It may seem as though it takes a force to keep an object moving. Push your book across a table—when you stop pushing, it stops moving.

But now, throw a ball across the room. The ball keeps moving after you let it go, even though you are not pushing it any more. Why?

It doesn’t take a force to keep an object moving in a straight line—it takes a force to change its motion. Your book stops because the force of friction stops it.
This is Newton’s first law, which is often called the law of inertia:

Every object continues in its state of rest, or of uniform velocity in a straight line, as long as no net force acts on it.

Conceptual Example 4-1: Newton’s first law.

A school bus comes to a sudden stop, and all of the backpacks on the floor start to slide forward. What force causes them to do that?

There is no force. By Newton's first law the backpacks continue their state of motion, maintaining their velocity. They will be brought to rest by friction or collision.
4-2 Newton’s First Law of Motion

**Inertial reference frames:**

Newton’s first law does not hold in every reference frame, such as a reference frame that is accelerating or rotating.

An inertial reference frame is one in which Newton’s first law is valid. This excludes rotating and accelerating frames.

How can we tell if we are in an inertial reference frame? By checking to see if Newton’s first law holds!

4-3 Mass

**Mass** is the measure of inertia of an object, sometimes understood as the quantity of matter in the object. In the SI system, mass is measured in kilogram (kg).

**Mass is not weight.**

**Mass is a property** of an object. Weight is the force exerted on that object by gravity.

If you go to the Moon, whose gravitational acceleration is about \( \frac{1}{6} g \), you will weigh much less. Your mass, however, will be the same.
Newton’s Second Law of Motion

Newton’s second law is the relation between acceleration and force. Acceleration is proportional to force and inversely proportional to mass.

\[ \sum F = ma \]

It takes a force to change either the direction or the speed of an object. More force means more acceleration; the same force exerted on a more massive object will yield less acceleration.

Newton’s Second Law – A Correction

Newton expressed his 2nd law in terms of momentum which is not discussed in Giancoli until chapter 9.

Momentum is a vector symbolized by the symbol \( \vec{p} \), and is defined as

\[ \vec{p} = m\vec{v} \]

Unit: kg.m/s

The correct statement of Newton’s 2nd law is:

The time rate of change of momentum of a body is proportional to the resultant external force acting on the body and takes place in the direction of the force.
Newton’s Second Law:

\[ \vec{F} \propto \frac{d \vec{p}}{dt} \]

In SI Units the constant is defined as 1 so we can write:

\[
\vec{F} = \frac{d \vec{p}}{dt} = \frac{d (m \vec{v})}{dt} = m \frac{d \vec{v}}{dt} + \vec{v} \frac{dm}{dt}
\]

Since \( \vec{a} = \frac{d \vec{v}}{dt} \) and in the special case where the mass of the body is constant

We can use \( \vec{F} = m \vec{a} \)

Do not try to solve variable mass problems, eg. Rocket motion without using the full form of Newton’s Second Law.

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**4-4 Newton’s Second Law of Motion**

Force is a vector, so \( \Sigma \vec{F} = ma \) is true along each coordinate axis.

\[
\Sigma F_x = ma_x, \quad \Sigma F_y = ma_y, \quad \Sigma F_z = ma_z
\]

where \( \vec{F} = F_x \hat{i} + F_y \hat{j} + F_z \hat{k} \)
The unit of force in the SI system is the newton (N).

Note that the pound is a unit of force, not of mass, and can therefore be equated to newtons but not to kilograms.

Since \( \sum F = ma \)

\[ 1 \text{ N} \equiv 1 \text{ kg.m/s}^2 \]

### 4-4 Newton’s Second Law of Motion

Example 4-2: Force to accelerate a fast car.

Estimate the net force needed to accelerate
(a) a 1000 kg car at \( \frac{1}{2}g \)
(b) a 200 g apple at the same rate.

(a) \( m = 1000 \text{ kg} \) \quad \( \ddot{a} = \frac{g}{2} = \frac{9.8}{2} = 4.9 \text{ m/s}^2 \)
\[
\sum \vec{F} = m\ddot{a} = 1000 \times 4.9 = 4900 \text{ N} = 4.9 \text{ kN}
\]

(b) \( m = 200 \text{ g} \) \quad \( \ddot{a} = \frac{g}{2} = \frac{9.8}{2} = 4.9 \text{ m/s}^2 \)
\[
\sum \vec{F} = m\ddot{a} = 0.2 \times 4.9 = 0.98 \text{ N}
\]
Example 4-3: Force to stop a car.

What average net force is required to bring a 1500 kg car to rest from a speed of 100 km/h within a distance of 55 m?

\[
100 \text{ km/h} = \frac{100 \times 1000}{3600} = 27.8 \text{ m/s}
\]

\[
v_0 = 100 \text{ km/h}
\]
\[
\begin{align*}
x & = 0 \\
x & = 55 \text{ m}
\end{align*}
\]

\[
\sum F = ma = -1500 \times 7.0 = -1.1 \times 10^4 \text{ N}
\]

Note that the force (and acceleration) is negative which shows that the force is in the opposite direction to the initial velocity.
4-5 Newton’s Third Law of Motion

Any time a force is exerted on an object, that force is caused by another object.

Newton's third law:

Whenever one object exerts a force on a second object, the second exerts an equal force in the opposite direction on the first.

Often stated as for every action there is an equal and opposite reaction.

Action and reaction forces act on different objects.

A key to the correct application of the third law is that the forces are exerted on different objects. Make sure you don’t use them as if they were acting on the same object.
4-5 Newton’s Third Law of Motion

Rocket propulsion can also be explained using Newton’s third law: hot gases from combustion spew out of the tail of the rocket at high speeds. The reaction force is what propels the rocket.

Note that the rocket does not need anything to “push” against.

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4-5 Newton’s Third Law of Motion

Conceptual Example 4-4: What exerts the force to move a car?

Response: A common answer is that the engine makes the car move forward. But it is not so simple. The engine makes the wheels go around. But if the tires are on slick ice or deep mud, they just spin. Friction is needed. On firm ground, the tires push backward against the ground because of friction. By Newton’s third law, the ground pushes on the tires in the opposite direction, accelerating the car forward.
4-5 Newton’s Third Law of Motion

Helpful notation: the first subscript is the object that the force is being exerted on; the second is the source.

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\[ \vec{F}_{GP} = -\vec{F}_{PG} \]
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Michelangelo’s assistant has been assigned the task of moving a block of marble using a sled. He says to his boss, “When I exert a forward force on the sled, the sled exerts an equal and opposite force backward. So how can I ever start it moving? No matter how hard I pull, the backward reaction force always equals my forward force, so the net force must be zero. I’ll never be able to move this load.” Is he correct?

Conceptual Example 4-5: Third law clarification.
Conceptual Example 4-5: Third law clarification.

Action–reaction forces do not cancel because they act on different objects.

Answer: No – in order to see whether the sled will accelerate, we need to consider only the forces on the sled. The force that the sled exerts on the assistant is irrelevant to the sled’s acceleration.

4-6 Weight—the Force of Gravity; and the Normal Force

Weight is the force exerted on an object by gravity. Close to the surface of the Earth, where the gravitational force is nearly constant, the weight of an object of mass $m$ is:

$$\mathbf{F}_G = mg,$$

where $g = 9.80 \text{ m/s}^2$.

Note that $g$ varies with position on the earth’s surface and with altitude.

$g$ varies from about 9.78 m/s$^2$ at the Equator to about 9.83 m/s$^2$ at the poles, so an object will weigh about 0.5% more at the poles than at the Equator.
An object at rest must have no net force on it. If it is sitting on a table, the force of gravity is still there; what other force is there?

The force exerted perpendicular to a surface is called the normal force. It is exactly as large as needed to balance the force from the object. (If the required force gets too big, something breaks!)

(a) The net force on an object at rest is zero according to Newton’s second law. Therefore the downward force of gravity (\(\vec{F}_G\)) on an object at rest must be balanced by an upward force (the normal force \(\vec{F}_N\)) exerted by the table in this case.

(b) \(\vec{F}_N\) is the force exerted on the table by the statue and is the reaction force to \(\vec{F}_N\) by Newton’s third law.
(a) Determine the weight of the box and the normal force exerted on it by the table.

Weight of box: \( \vec{W} = m\vec{g} = 10.0 \times 9.80 = 98.0 \text{ N} \)

\[ \sum F_y = F_N - mg = 0 \quad \rightarrow \quad F_N = mg = 98.0 \text{ N upwards} \]

(b) Now your friend pushes down on the box with a force of 40.0 N. Again determine the normal force exerted on the box by the table.

\[ \sum F_y = F_N - mg - 40.0 = 0 \]

\[ F_N = mg + 40.0 = 98.0 + 40.0 = 138.0 \text{ N upwards}. \]
A friend has given you a special gift, a box of mass 10.0 kg with a mystery surprise inside. The box is resting on the smooth (frictionless) horizontal surface of a table.

(c) If your friend pulls upward on the box with a force of 40.0 N, what now is the normal force exerted on the box by the table?

\[
\sum F_y = F_N - mg + 40.0 = 0
\]

\[
F_N = mg - 40.0 = 98.0 - 40.0 = 58.0 \text{ N upwards.}
\]

Example 4-7: Accelerating the box.

What happens when a person pulls upward on the box in the previous example with a force greater than the box’s weight, say 100.0 N?
Example 4-8: Apparent weight loss.

A 65 kg woman descends in an elevator that briefly accelerates at $0.20g$ downward. She stands on a scale that reads in kg.

(a) During this acceleration, what is her weight and what does the scale read?

\[ \Sigma F_y = F_p - mg = ma_y \]
\[ a_y = \frac{F_p - mg}{m} = \frac{100.0 - 98.0}{10.0} = 0.20 \text{ m/s}^2 \] upwards
Weight is always $mg$ and will not change when the elevator accelerates

$W = mg = 65 \times 9.8 = 640 \text{ N}$

Strictly scales measure force but are calibrated to show the mass of the person → would read 65 kg if the elevator was not moving.

When the elevator is accelerating downwards (take ↓ as positive):

$$\sum F = mg - F_N = ma$$

$$F_N = mg - ma = m(g - a) = m(g - 0.2g) = 0.8mg$$

Scale reading $= 0.8 \times 65 = 52 \text{ kg}$

Example 4-8: Apparent weight loss.

A 65 kg woman descends in an elevator that briefly accelerates at $0.20g$ downward. She stands on a scale that reads in kg.

(b) What does the scale read when the elevator descends at a constant speed of 2.0 m/s?

Without acceleration

$$\sum F = mg - F_N = 0 \quad \rightarrow \quad F_N = mg$$

and the scale reads the correct mass of 65 kg
4-7 Solving Problems with Newton’s Laws: Free-Body Diagrams

1. Draw a sketch.

2. For one object, draw a free-body diagram, showing all the forces acting on the object. Make the magnitudes and directions as accurate as you can. Label each force. If there are multiple objects, draw a separate diagram for each one.

3. Resolve vectors into components.

4. Apply Newton’s second law to each component.

5. Solve.

### Example:

\[
\begin{align*}
F_{Ax} &= F_A \cos 45.0^\circ = 40.0 \times 0.707 = 28.3 \text{ N} \\
F_{Ay} &= F_A \sin 45.0^\circ = 40.0 \times 0.707 = 28.3 \text{ N} \\
F_{Bx} &= F_B \cos 37.0^\circ = 30.0 \times 0.799 = 24.0 \text{ N} \\
F_{By} &= -F_B \sin 37.0^\circ = -30.0 \times 0.602 = -18.1 \text{ N} \\
F_{Rx} &= F_{Ax} + F_{Bx} = 28.3 + 24.0 = 52.3 \text{ N} \\
F_{ Ry } &= F_{Ay} + F_{By} = 28.3 - 18.1 = 10.2 \text{ N} \\
F_R &= \sqrt{F_{Rx}^2 + F_{Ry}^2} = \sqrt{52.3^2 + 10.2^2} = 53.3 \text{ N} \\
\theta &= \tan^{-1} \frac{F_{Ry}}{F_{Rx}} = \tan^{-1} \frac{10.2}{52.3} = 11.0^\circ
\end{align*}
\]

Net force = 53.3 N at 11.0° to x axis
By convention angles are measured relative to the x axis:
Anti-clockwise (acw) is +ve
Clockwise (cw) is –ve

\[ \theta_A = 34^\circ \]
\[ \theta_B = -56^\circ = +304^\circ \]
\[ \theta_C = 144^\circ \]
Example 4-11: Pulling the mystery box. Suppose a friend asks to examine the 10.0 kg box you were given previously, hoping to guess what is inside; and you respond, “Sure, pull the box over to you.” She then pulls the box by the attached cord along the smooth surface of the table. The magnitude of the force exerted by the person is $F_P = 40.0 \text{ N}$, and it is exerted at a $30.0^\circ$ angle as shown.

Calculate
(a) the acceleration of the box, and
(b) the magnitude of the upward force $F_N$ exerted by the table on the box.

(a) the acceleration of the box

\[
F_{P_x} = F_P \cos 30^\circ = 40.0 \times 0.866 = 34.6 \text{ N}
\]

\[
F_{P_x} = ma_x \rightarrow a_x = \frac{F_{P_x}}{m} = \frac{34.6}{10.0} = 3.46 \text{ m/s}^2 \text{ to right}
\]

(b) the magnitude of the upward force $F_N$ exerted by the table on the box.

\[
F_{P_y} = F_P \sin 30^\circ = 40.0 \times 0.500 = 20.0 \text{ N}
\]

\[
F_N + F_{P_y} - mg = 0
\]

\[
F_N = mg - F_{P_y} = 10.0 \times 9.80 - 20.0 = 78.0 \text{ N}
\]
Example 4-12: Two boxes connected by a cord.

Two boxes, A and B, are connected by a lightweight cord and are resting on a smooth table. The boxes have masses of 12.0 kg and 10.0 kg. A horizontal force of 40.0 N is applied to the 10.0-kg box. Find (a) the acceleration of each box, and (b) the tension in the cord connecting the boxes.

(a) the acceleration of each box

Vertical forces cancel so only need to consider horizontal forces.

Both boxes have the same acceleration.

Box A: \[ \Sigma F_A = F_p - F_T = m_A a \]
Box B: \[ \Sigma F_B = F_T = m_B a \]

Add equations to eliminate \( F_T \)

\[ F_p - F_T + F_T = m_A a + m_B a = a(m_A + m_B) \]

\[ a = \frac{F_p}{m_A + m_B} = \frac{40.0}{10.0 + 12.0} = 1.82 \text{ m/s}^2 \]
(b) the tension in the cord connecting the boxes.

Only need to consider one box.

Box B: \[ F_T = m_Ba = 12.0 \times 1.82 = 21.8 \text{ N} \]
or Box A (gives the same answer)

\[ F_p - F_T = m_Aa \rightarrow F_T = F_p - m_Aa \]
\[ = 40.0 - 10.0 \times 1.82 = 21.8 \text{ N} \]
Example 4-13: Elevator and counterweight (Atwood's machine).

(a) the acceleration of the elevator

Elevator: \[ F_T - m_E g = -m_E a_E \]

Counterweight: \[ F_T - m_C g = m_C a_C \]
\[ \ddot{a}_E = -\ddot{a}_C \rightarrow |\ddot{a}_E| = |\ddot{a}_C| = a \]
\[ F_T = m_E g - m_E a = m_C g + m_C a \]
\[ a = \frac{g(m_E - m_C)}{m_E + m_C} = \frac{9.80(1150 - 1000)}{1150 + 1000} = 0.68 \text{ m/s}^2 \]

(b) the tension in the cable.
Consider Elevator:
\[ F_T - m_E g = -m_E a \]
\[ F_T = m_E (g - a) = 1150(9.80 - 0.68) = 10,500 \text{ N} \]

Consider Counterweight gives the same answer
\[ F_T - m_C g = -m_C a \]
\[ F_T = m_C (g + a) = 1000(9.80 + 0.68) = 10,500 \text{ N} \]
4-7 Solving Problems with Newton’s Laws: Free-Body Diagrams

Conceptual Example 4-14: The advantage of a pulley.

A mover is trying to lift a piano (slowly) up to a second-story apartment. He is using a rope looped over two pulleys as shown. What force must he exert on the rope to slowly lift the piano’s 2000 N weight?

To lift the piano at constant speed $\vec{a}_y = 0$

$\Sigma F_y = F_T + F_T - mg = ma_y = 0$

$2F_T = mg \rightarrow F_T = \frac{mg}{2} = \frac{2000}{2} = 1000 \text{ N}$

Example 4-15: Accelerometer.

A small mass $m$ hangs from a thin string and can swing like a pendulum. You attach it above the window of your car as shown. What angle does the string make (a) when the car accelerates at a constant $a = 1.20 \text{ m/s}^2$, and (b) when the car moves at constant velocity, $v = 90 \text{ km/h}$?
What angle does the string make (a) when the car accelerates at a constant $a = 1.20 \, \text{m/s}^2$, and (b) when the car moves at constant velocity, $v = 90 \, \text{km/h}$?

Taking $\vec{a}$ & $\vec{v}$

(a) Horizontally: $F_x = F_x \sin \theta = ma$

Vertically: $F_y = F_y \cos \theta - mg = 0$

$\sin \theta = \frac{ma}{F_x}$ \quad (1) \qquad \cos \theta = \frac{mg}{F_x}$ \quad (2)

divide (1) by (2) \qquad \frac{\sin \theta}{\cos \theta} = \tan \theta = \frac{a}{g}$

$\theta = \tan^{-1} \frac{a}{g} = \tan^{-1} \frac{1.20}{9.80} = 7.0^\circ$

(b) At constant velocity $a = 0 \rightarrow \theta = 0^\circ$

Example 4-16: Box slides down an incline.

A box of mass $m$ is placed on a smooth incline that makes an angle $\theta$ with the horizontal.

(a) Determine the normal force on the box. (b) Determine the box’s acceleration. (c) Evaluate for a mass $m = 10 \, \text{kg}$ and an incline of $\theta = 30^\circ$. 

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4-7 Solving Problems with Newton’s Laws: Free-Body Diagrams

(a) Normal force on the box.

\[ \sum F_y = F_N = mg \cos \theta = 0 \]

\[ F_N = mg \cos \theta \]

(b) Box’s acceleration.

\[ \sum F_x = mg \sin \theta = ma \]

\[ a = g \sin \theta \]

(c) Evaluate \( F_N \) and \( a \) for \( m = 10 \text{ kg} \) and \( \theta = 30^\circ \).

\[ F_N = 10.0 \times 9.80 \cos 30^\circ = 85 \text{ N} \]

\[ a = 9.80 \sin 30^\circ = 4.9 \text{ m/s}^2 \]

4-8 Problem Solving—A General Approach

1. Read the problem carefully; then read it again.
2. Draw a sketch, and then a free-body diagram.
3. Choose a convenient coordinate system.
4. List the known and unknown quantities; find relationships between the knowns and the unknowns.
5. Estimate the answer.
6. Solve the problem without putting in any numbers (algebraically); once you are satisfied, put the numbers in.
7. Keep track of dimensions.
8. Make sure your answer is reasonable.
Summary of Chapter 4

• Newton’s first law: If the net force on an object is zero, it will remain either at rest or moving in a straight line at constant speed.

• Newton’s second law:  \( \sum \vec{F} = m\vec{a} \).

• Newton’s third law:  \( \vec{F}_{AB} = -\vec{F}_{BA} \).

• Weight is the gravitational force on an object.

• Free-body diagrams are essential for problem-solving. Do one object at a time, make sure you have all the forces, pick a coordinate system and find the force components, and apply Newton’s second law along each axis.