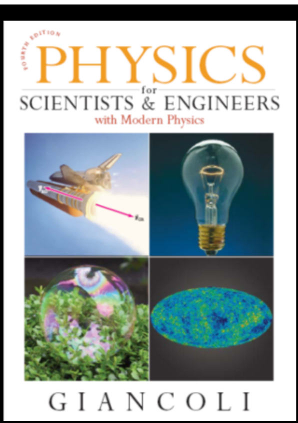


Lecture PowerPoints

Chapter 11

Physics for Scientists and Engineers, with Modern Physics, 4th edition

Giancoli

Chapter 11

Angular Momentum; General Rotation

Units of Chapter 11

- Angular Momentum—Objects Rotating About a Fixed Axis
- Vector Cross Product; Torque as a Vector
- Angular Momentum of a Particle
- Angular Momentum and Torque for a System of Particles; General Motion
- Angular Momentum and Torque for a Rigid Object
- Conservation of Angular Momentum
- The Spinning Top and Gyroscope

11-1 Angular Momentum—Objects Rotating About a Fixed Axis

The rotational analog of linear momentum is angular momentum, L:

$$L = I\omega.$$

Then the rotational analog of Newton's second law is:

$$\Sigma\tau = \frac{dL}{dt}.$$

This form of Newton's second law is valid even if I is not constant.

11-1 Angular Momentum—Objects Rotating About a Fixed Axis

In the absence of an external torque, angular momentum is conserved:

$$\frac{dL}{dt} = 0 \text{ and } L = I\omega = \text{constant.}$$

More formally,

the total angular momentum of a rotating object remains constant if the net external torque acting on it is zero.

11-1 Angular Momentum—Objects Rotating About a Fixed Axis

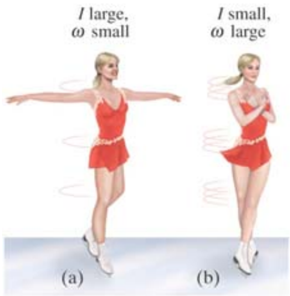
This means:

$$I\omega = I_0\omega_0 = \text{constant.}$$

Therefore, if an object's moment of inertia changes, its angular speed changes as well.

11-1 Angular Momentum—Objects Rotating About a Fixed Axis

$$I\omega = I_0\,\omega_0 = \text{constant.}$$



A skater doing a spin on ice, illustrating conservation of angular momentum: (a) I is large and ω is small; (b) I is smaller so ω is larger.
A diver rotates faster when arms and legs are tucked in than when they are outstretched.

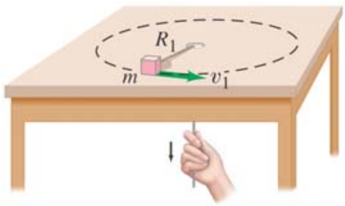
Conservation of Angular Momentum

Click on frame to show video

11-1 Angular Momentum—Objects Rotating About a Fixed Axis

Example 11-1: Object rotating on a string of changing length.

A small mass m attached to the end of a string revolves in a circle on a frictionless tabletop. The other end of the string passes through a hole in the table. Initially, the mass revolves with a speed v_1 = 2.4 m/s in a circle of radius R_1 = 0.80 m. The string is then pulled slowly through the hole so that the radius is reduced to R_2 = 0.48 m. What is the speed, v_2, of the mass now?



11-1 Angular Momentum—Objects Rotating About a Fixed Axis

For small mass: $I = mr^2$

Conservation of Angular Momentum: $I\omega = I_0\omega_0$

$$\therefore mr^2\omega = mr_0^2\omega_0 \rightarrow r^2\omega = r_0^2\omega_0$$

$$\& \; v = r\omega \quad \rightarrow \quad \omega = \frac{v}{r}$$

$$\therefore \frac{r^2 v}{r} = \frac{r_0^2 v_0}{r_0} \quad \rightarrow \quad rv = r_0 v_0$$

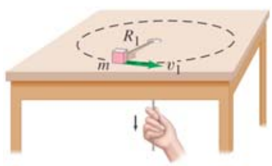
$$\therefore v_2 = v = \frac{r_0 v_0}{r} = \frac{0.80 \times 2.4}{0.48} = 4.0 \text{ m/s}$$

$v_0 = v_1 = 2.4$ m/s

$v = v_2$

$r_0 = R_1 = 0.80$ m

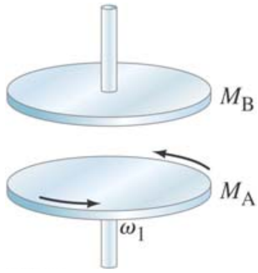
$r = R_2 = 0.48$ m



11-1 Angular Momentum—Objects Rotating About a Fixed Axis

Example 11-2: Clutch.

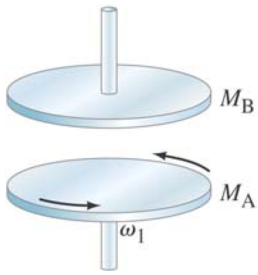
A simple clutch consists of two cylindrical plates that can be pressed together to connect two sections of an axle, as needed, in a piece of machinery. The two plates have masses M_A = 6.0 kg and M_B = 9.0 kg, with equal radii R_0 = 0.60 m. They are initially separated. Plate M_A is accelerated from rest to an angular velocity ω_1 = 7.2 rad/s in time Δt = 2.0 s. Calculate (c) Next, plate M_B, initially at rest but free to rotate without friction, is placed in firm contact with freely rotating plate M_A, and the two plates both rotate at a constant angular velocity ω_2, which is considerably less than ω_1. Why does this happen, and what is ω_2?



11-1 Angular Momentum—Objects Rotating About a Fixed Axis

Example 11-2: Clutch.

A simple clutch consists of two cylindrical plates that can be pressed together to connect two sections of an axle, as needed, in a piece of machinery. The two plates have masses M_A = 6.0 kg and M_B = 9.0 kg, with equal radii R_0 = 0.60 m. They are initially separated. Plate M_A is accelerated from rest to an angular velocity ω_1 = 7.2 rad/s in time Δt = 2.0 s. Calculate (a) the angular momentum of M_A



$$L = I\omega$$

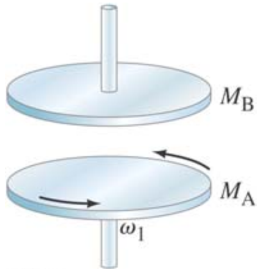
$$I_A = \tfrac{1}{2} M_A R_A^2 = \frac{6.0 \times 0.6^2}{2} = 1.08\ \text{kg.m}^2$$

$$\therefore L = I\omega = 1.08 \times 7.2 = 7.8\ \text{kg.m}^2/\text{s}$$

11-1 Angular Momentum—Objects Rotating About a Fixed Axis

Example 11-2: Clutch.

A simple clutch consists of two cylindrical plates that can be pressed together to connect two sections of an axle, as needed, in a piece of machinery. The two plates have masses M_A = 6.0 kg and M_B = 9.0 kg, with equal radii R_0 = 0.60 m. They are initially separated. Plate M_A is accelerated from rest to an angular velocity ω_1 = 7.2 rad/s in time Δt = 2.0 s. Calculate (b) the torque required to have accelerated M_A from rest to ω_1



$$\tau = I\alpha \quad \& \; \omega = \omega_0 + \alpha t$$

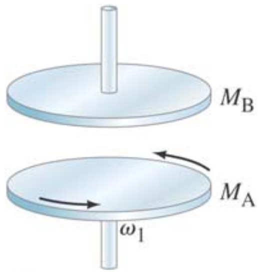
$$\therefore \alpha = \frac{\omega - \omega_0}{\Delta t} = \frac{7.2 - 0}{2.0} = 3.60 \text{ rad/s}^2$$

$$\therefore \tau = I\alpha = 7.8 \times 3.60 = 3.9 \text{ N.m}$$

11-1 Angular Momentum—Objects Rotating About a Fixed Axis

Example 11-2: Clutch.

Calculate (c) plate M_B, initially at rest but free to rotate without friction, is placed in firm contact with freely rotating plate M_A, and the two plates both rotate at a constant angular velocity ω_2, which is considerably less than ω_1. Why does this happen, and what is ω_2?



Angular momentum is conserved

(this is a rotational collision)

$$I_A = \tfrac{1}{2} M_A R_A^2 = \frac{6.0 \times 0.6^2}{2} = 1.08 \text{ kg.m}^2$$

$$I_B = \tfrac{1}{2} M_B R_B^2 = \frac{9.0 \times 0.6^2}{2} = 1.62 \text{ kg.m}^2$$

$$I_A \omega_A = (I_A + I_B)\omega_B$$

$$\omega_B = \frac{I_A \omega_A}{I_A + I_B} = \frac{1.08 \times 7.2}{1.08 + 1.62} = 2.9 \text{ rad/s}$$

11-1 Angular Momentum—Objects Rotating About a Fixed Axis

Example 11-3: Neutron star.

Astronomers detect stars that are rotating extremely rapidly, known as neutron stars. A neutron star is believed to form from the inner core of a larger star that collapsed, under its own gravitation, to a star of very small radius and very high density. Before collapse, suppose the core of such a star is the size of our Sun ($r \approx 7 \times 10^5$ km) with mass 2.0 times as great as the Sun, and is rotating at a frequency of 1.0 revolution every 100 days. If it were to undergo gravitational collapse to a neutron star of radius 10 km, what would its rotation frequency be? Assume the star is a uniform sphere at all times, and loses no mass.

11-1 Angular Momentum—Objects Rotating About a Fixed Axis

Example 11-3: Neutron star.

$$\begin{aligned} r_0 &= 7\times10^5\ \text{km} \\ \omega_0 &= 1\,\text{rev}/100\,\text{days} \\ &= \frac{1}{100\times24\times3600} = 1.16\times10^{-7}\ \text{rev/s} \\ r &= 10\,\text{km} \end{aligned}$$

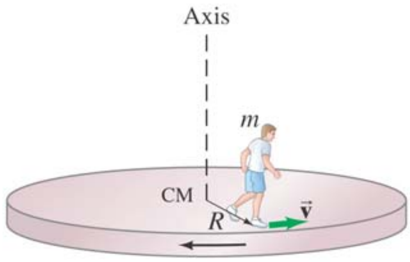
$$I\omega = I_0\omega_0$$

$$\omega = \frac{I_0\omega_0}{I} = \left(\frac{\frac{2}{5}mr_0^2}{\frac{2}{5}mr^2}\right)\omega_0 = \left(\frac{r_0}{r}\right)^2\omega_0$$

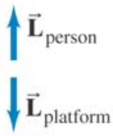
$$= \left(\frac{7\times10^5}{10}\right)^2 \times 1.16\times10^{-7} = 568 \approx 600\ \text{rev/s}$$

11-1 Angular Momentum—Objects Rotating About a Fixed Axis

Angular momentum is a vector; for a symmetrical object rotating about a symmetry axis it is in the same direction as the angular velocity vector.



Angular momentum points along the axis of rotation in a direction given by the right hand rule. – Fingers curl in the direction of rotation – Thumb points in direction of angular momentum vector



11-1 Angular Momentum—Objects Rotating About a Fixed Axis

Example 11-4: Running on a circular platform.

Suppose a 60 kg person stands at the edge of a 6.0 m diameter circular platform, which is mounted on frictionless bearings and has a moment of inertia of 1800 kg·m^2. The platform is at rest initially, but when the person begins running at a speed of 4.2 m/s (with respect to the Earth) around its edge, the platform begins to rotate in the opposite direction. Calculate the angular velocity of the platform.

Example 11-4: Running on a circular platform.

$v = r\omega \rightarrow \omega = v/r$

$m = 60\text{ kg}$

$r = 3.0\text{ m}$

$v = 4.2\text{ m/s}$

$I_{\text{person}} = mr^2$

$I_{\text{platform}} = 1800\text{ kg.m}^2$

$L_{\text{person}} = I_{\text{person}}\omega = mr^2(v/r) = mrv$

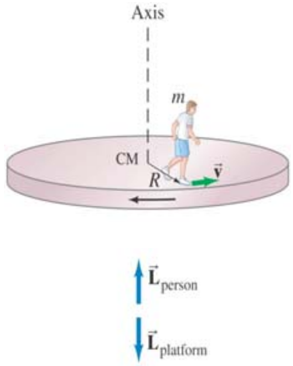
Conservation of Angular Momentum

Total $\vec{\mathbf{L}}$ is zero

$L = L_{\text{person}} + L_{\text{platform}}$

$0 = mrv - I_{\text{platform}}\omega$

$$\omega = \frac{mrv}{I_{\text{platform}}} = \frac{60 \times 3.0 \times 0.42}{1800} = 0.42\text{ rad/s}$$



Note: $\omega = 2\pi f = 2\pi/T \rightarrow T = 2\pi/\omega = 15\text{ s per revolution}$

11-1 Angular Momentum—Objects Rotating About a Fixed Axis

Conceptual Example 11-5: Spinning bicycle wheel.

Your physics teacher is holding a spinning bicycle wheel while he stands on a stationary frictionless turntable. What will happen if the teacher suddenly flips the bicycle wheel over so that it is spinning in the opposite direction?

Conceptual Example 11-5: Spinning bicycle wheel.

Angular momentum is conserved, so the person will start spinning in the direction the wheel was spinning originally.

Click image to play video

Initially, we have

$L_{system} = L_0$ (upward)

After the wheel is inverted,

$L_{system} = L_0 = L_{person+turntable} + L_{wheel}$

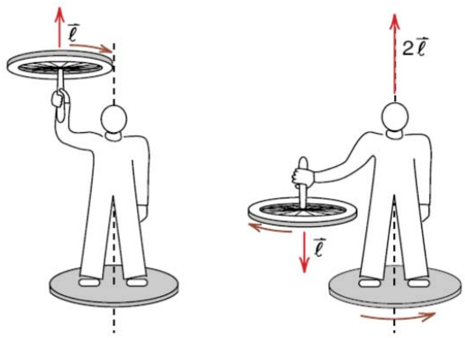
In this case $L_{wheel} = -L_0$

(rotating in opposite direction)

$\therefore\ L_0 = L_{person+turntable} - L_0$

$\therefore\ L_{person+turntable} = 2L_0$

This shows that the person and turntable will rotate in the same direction as the original direction of the wheel and with an angular momentum equal to twice the initial angular momentum of the wheel.

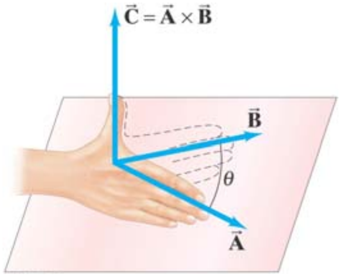


11-2 Vector Cross Product; Torque as a Vector

The vector cross product is defined as:

$$C = \left|\vec{\mathbf{A}} \times \vec{\mathbf{B}}\right| = AB\sin\theta.$$

The direction of the cross product is defined by a right-hand rule:



11-2 Vector Cross Product; Torque as a Vector

The cross product can also be written in determinant form:

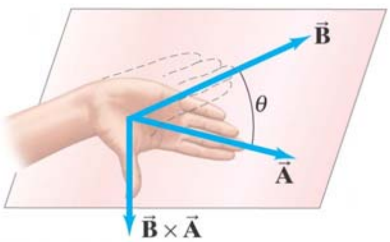
$$\vec{\mathbf{A}} \times \vec{\mathbf{B}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$= (A_y B_z - A_z B_y)\hat{\mathbf{i}} + (A_z B_x - A_x B_z)\hat{\mathbf{j}} + (A_x B_y - A_y B_x)\hat{\mathbf{k}}.$$

11-2 Vector Cross Product; Torque as a Vector

Some properties of the cross product:

$$\vec{\mathbf{A}} \times \vec{\mathbf{A}} = 0$$
$$\vec{\mathbf{A}} \times \vec{\mathbf{B}} = -\vec{\mathbf{B}} \times \vec{\mathbf{A}}$$
$$\vec{\mathbf{A}} \times (\vec{\mathbf{B}} + \vec{\mathbf{C}}) = (\vec{\mathbf{A}} \times \vec{\mathbf{B}}) + (\vec{\mathbf{A}} \times \vec{\mathbf{C}})$$
$$\frac{d}{dt}(\vec{\mathbf{A}} \times \vec{\mathbf{B}}) = \frac{d\vec{\mathbf{A}}}{dt} \times \vec{\mathbf{B}} + \vec{\mathbf{A}} \times \frac{d\vec{\mathbf{B}}}{dt}.$$

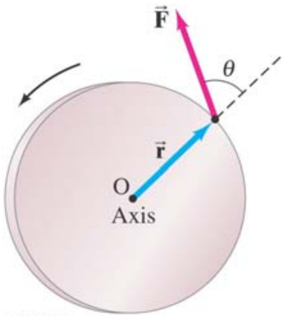


11-2 Vector Cross Product; Torque as a Vector

Torque can be defined as the vector product of the force and the vector from the point of action of the force to the axis of rotation:

$$\vec{\boldsymbol{\tau}} = \vec{\mathbf{r}} \times \vec{\mathbf{F}}.$$

Using the right hand rule shows that the torque vector points along the axis of rotation and is out of the page.

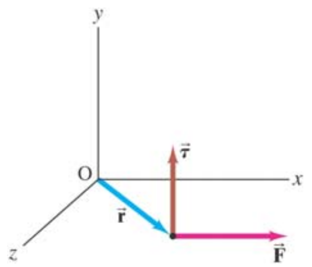


11-2 Vector Cross Product; Torque as a Vector

For a particle, the torque can be defined around a point O:

$$\vec{\tau} = \vec{\mathbf{r}} \times \vec{\mathbf{F}}.$$

Here, $\vec{\mathbf{r}}$ is the position vector from the particle relative to O.



Example 11-6: Torque vector.

Suppose the vector $\vec{\mathbf{r}}$ is in the xz plane, and is given by $\vec{\mathbf{r}}$ = (1.2 m) $\hat{\mathbf{i}}$ + 1.2 m) $\hat{\mathbf{k}}$. Calculate the torque vector $\vec{\tau}$ if $\vec{\mathbf{F}}$ = (150 N) $\hat{\mathbf{i}}$.

$$\vec{\boldsymbol{\tau}} = \vec{\mathbf{r}} \times \vec{\mathbf{F}} = \left(1.2\,\hat{\mathbf{i}} + 1.2\,\hat{\mathbf{k}}\right) \times 150\,\hat{\mathbf{i}}$$

$$= 180\,\hat{\mathbf{j}}\ \text{N.m}$$

ie. $\vec{\boldsymbol{\tau}}$ is in y direction

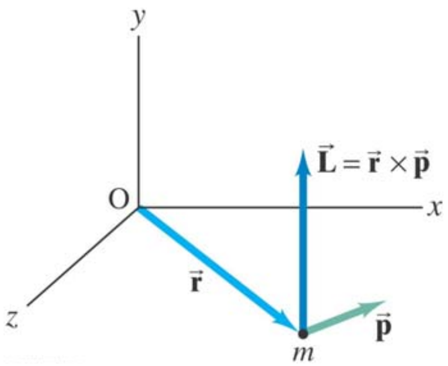
Note: $\hat{\mathbf{i}} \times \hat{\mathbf{i}} = 0$

$\hat{\mathbf{k}} \times \hat{\mathbf{i}} = \hat{\mathbf{j}}$

11-3 Angular Momentum of a Particle

The angular momentum of a particle about a specified axis is given by:

$$\vec{\mathbf{L}} = \vec{\mathbf{r}} \times \vec{\mathbf{p}}.$$



11-3 Angular Momentum of a Particle

If we take the derivative of $\vec{L}$, we find:

$$\frac{d\vec{\mathbf{L}}}{dt} = \vec{\mathbf{r}} \times \frac{d\vec{\mathbf{p}}}{dt}.$$

Since $\vec{\mathbf{r}} \times \Sigma\vec{\mathbf{F}} = \vec{\mathbf{r}} \times \frac{d\vec{\mathbf{p}}}{dt} = \frac{d\vec{\mathbf{L}}}{dt},$

we have: $\Sigma\vec{\boldsymbol{\tau}} = \frac{d\vec{\mathbf{L}}}{dt}.$

This is the rotational analogue of Newton's 2nd Law

11-3 Angular Momentum of a Particle

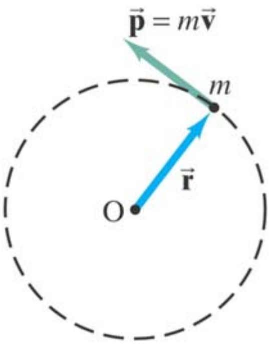
Conceptual Example 11-7: A particle's angular momentum.

What is the angular momentum of a particle of mass *m* moving with speed *v* in a circle of radius *r* in a counterclockwise direction?

$$\vec{\mathbf{L}} = \vec{\mathbf{r}} \times \vec{\mathbf{p}} = \vec{\mathbf{r}} \times m\vec{\mathbf{v}}$$

$\vec{\mathbf{L}}$ is directed outwards perpendicular to both $\vec{\mathbf{r}}$ and $\vec{\mathbf{v}}$ (RH Rule)

$$L = rmv \sin 90° = rm.r\omega = mr^2\omega = I\omega$$



11-4 Angular Momentum and Torque for a System of Particles; General Motion

The angular momentum of a system of particles can change only if there is an external torque—torques due to internal forces cancel.

$$\frac{d\vec{\mathbf{L}}}{dt} = \sum \vec{\boldsymbol{\tau}}_{\text{ext}}.$$

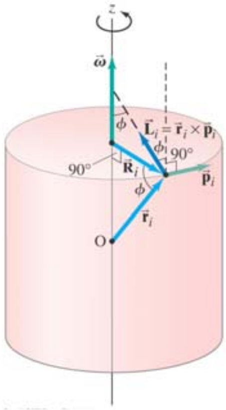
This equation is valid in any inertial reference frame. It is also valid for the center of mass, even if it is accelerating:

$$\frac{d\vec{\mathbf{L}}_{\text{CM}}}{dt} = \sum \vec{\boldsymbol{\tau}}_{\text{CM}}.$$

11-5 Angular Momentum and Torque for a Rigid Object

For a rigid object, we can show that its angular momentum when rotating around a particular axis is given by:

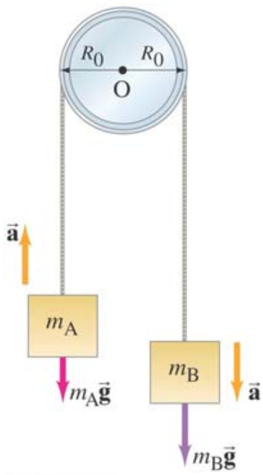
$$L_\omega = I\omega.$$



11-5 Angular Momentum and Torque for a Rigid Object

Example 11-8: Atwood's machine.

An Atwood machine consists of two masses, m_A and m_B, which are connected by an inelastic cord of negligible mass that passes over a pulley. If the pulley has radius R_0 and moment of inertia I about its axle, determine the acceleration of the masses m_A and m_B, and compare to the situation where the moment of inertia of the pulley is ignored.



The angular momentum about O is

$$L = L_{\text{pulley}} + L_{m_A} + L_{m_B}$$

$$= I\omega + R_0 m_A v + R_0 m_B v$$

Also $\omega = v/R_0$

$$L = (m_A + m_B)vR_0 + \frac{Iv}{R_0}$$

The external torque about O is

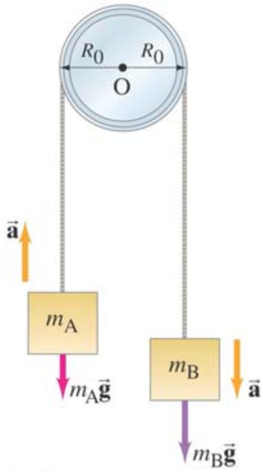
$$\tau = m_B g R_0 - m_A g R_0 \quad \text{(taking clockwise as + ve)}$$

$$\tau = \frac{dL}{dt}$$

$$(m_B - m_A)gR_0 = (m_A + m_B)R_0 \frac{dv}{dt} + \frac{I}{R_0}\frac{dv}{dt}$$

$$a = \frac{dv}{dt} = \frac{(m_B - m_A)g}{(m_A + m_B) + I/R_0^2}$$

Example 11-8: Atwood's machine.



$$a = \frac{dv}{dt} = \frac{(m_B - m_A)g}{(m_A + m_B) + I/R_0^2}$$

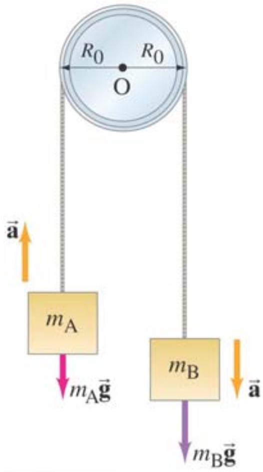
If the moment of inertia of the pulley is ignored we get

$$a = \frac{dv}{dt} = \frac{(m_B - m_A)g}{(m_A + m_B)}$$

Which is the same result as for the ideal acceleration in experiment E1 Mechanics and Forces where the effect of the pulley was ignored (pulley assumed to have negligible mass).

The effect of the moment of inertia of the pulley is to slow down the system.

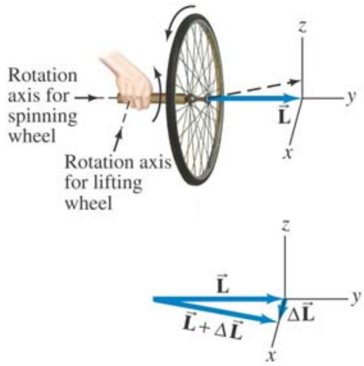
Example 11-8: Atwood's machine.



11-5 Angular Momentum and Torque for a Rigid Object

Conceptual Example 11-9: Bicycle wheel.

Suppose you are holding a bicycle wheel by a handle connected to its axle. The wheel is spinning rapidly so its angular momentum points horizontally as shown. Now you suddenly try to tilt the axle upward (so the CM moves vertically). You expect the wheel to go up (and it would if it weren't rotating), but it unexpectedly swerves to the right! Explain.



Conceptual Example 11-9: Bicycle wheel.

$$\vec{\tau}_{\text{net}} = \frac{d\vec{L}}{d\vec{t}}$$

In the short time Δt you exert a net torque (about an axis through your wrist)

that points along the x axis perpendicular to $\vec{L}$

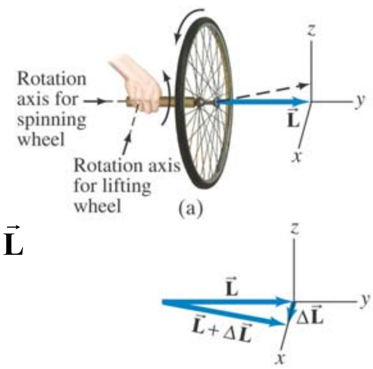
So $\Delta\vec{L} = \vec{\tau}_{\text{net}}\Delta t$

$\Delta\vec{L}$ points (approximately) along the x axis

Since $\vec{L}$ is directed along the axis of the wheel

the axle is now pointing in the direction of $\vec{L} + \Delta\vec{L}$

ie. the axle veers to the right



11-6 Conservation of Angular Momentum

If the net torque on a system is constant,

$$\frac{d\vec{\mathbf{L}}}{dt} = 0 \quad \text{and} \quad \vec{\mathbf{L}} = \text{constant.} \qquad \left[\Sigma\vec{\boldsymbol{\tau}} = 0\right]$$

The total angular momentum of a system remains constant if the net external torque acting on the system is zero.

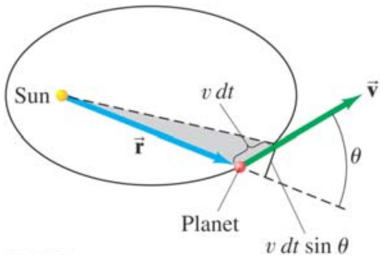
This is just the vector form of the result in section 11-1

NB: $L = I\omega$ is ONLY valid if the rotational axis is along a symmetry axis through the centre of mass.

11-6 Conservation of Angular Momentum

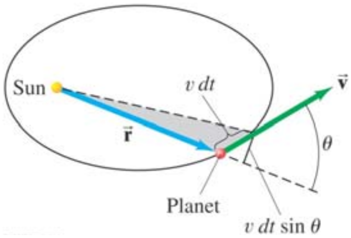
Example 11-11: Kepler's second law derived.

Kepler's second law states that each planet moves so that a line from the Sun to the planet sweeps out equal areas in equal times. Use conservation of angular momentum to show this.



Example 11-11: Kepler's second law derived.

In time dt the planet moves a distance vdt and sweeps out an area dA equal to the area of the triangle shown $dA = \frac{1}{2}(r)(vdt\sin\theta)$



$$\frac{dA}{dt} = \tfrac{1}{2} rv\sin\theta$$

The magnitude of the angular momentum about the Sun is

$$L = \left|\vec{\mathbf{r}} \times m\vec{\mathbf{v}}\right| = rmv\sin\theta \quad \rightarrow \quad rv\sin\theta = \frac{L}{m}$$

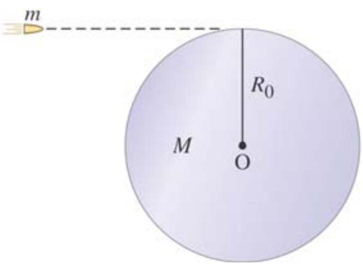
$$\frac{dA}{dt} = \tfrac{1}{2} rv\sin\theta = \frac{L}{2m}$$

Since the gravitational force $\vec{\mathbf{F}}$ is directed towards the Sun it produces no torque so $\vec{\mathbf{L}}$ is constant $\rightarrow \dfrac{dA}{dt}$ is constant

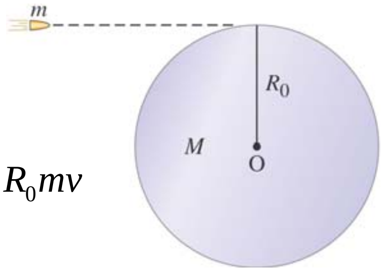
11-6 Conservation of Angular Momentum

Example 11-12: Bullet strikes cylinder edge.

A bullet of mass m moving with velocity v strikes and becomes embedded at the edge of a cylinder of mass M and radius R_0. The cylinder, initially at rest, begins to rotate about its symmetry axis, which remains fixed in position. Assuming no frictional torque, what is the angular velocity of the cylinder after this collision? Is kinetic energy conserved?



Example 11-12: Bullet strikes cylinder edge.



Initial $\vec{\mathbf{L}}_0$ is that of bullet only: $L_0 = R_0 mv$

Final $\vec{\mathbf{L}}$ is due to bullet + cylinder:

$I_B = mR_0^2 \qquad I_C = \frac{1}{2}MR_0^2$

$L = I\omega = (I_B + I_C)\omega = \left(mR_0^2 + \frac{1}{2}MR_0^2\right)\omega = \left(\frac{1}{2}M + m\right)R_0^2\omega$

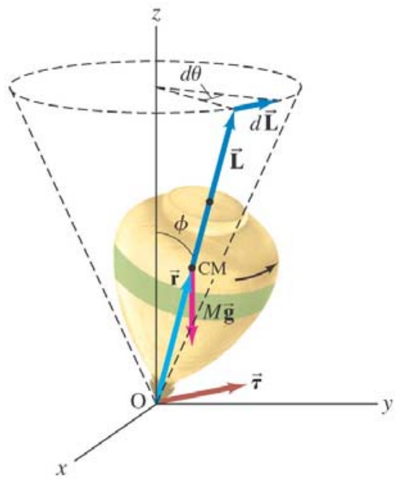
Applying conservation of angular momentum

$$\left(\frac{1}{2}M + m\right)R_0^2\omega = R_0 mv \rightarrow \omega = \frac{mv}{\left(\frac{1}{2}M + m\right)R_0}$$

This is an inelastic collision - Kinetic energy is not conserved

11-7 The Spinning Top and Gyroscope

A spinning top will precess around its point of contact with a surface, due to the torque created by gravity when its axis of rotation is not vertical.

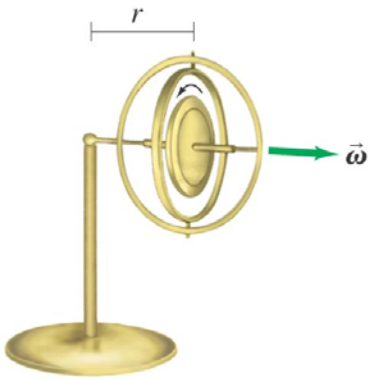


11-7 The Spinning Top and Gyroscope

The angular velocity of the precession is given by:

$$\Omega = \frac{Mgr}{I\omega}.$$

This is also the angular velocity of precession of a toy gyroscope, as shown.

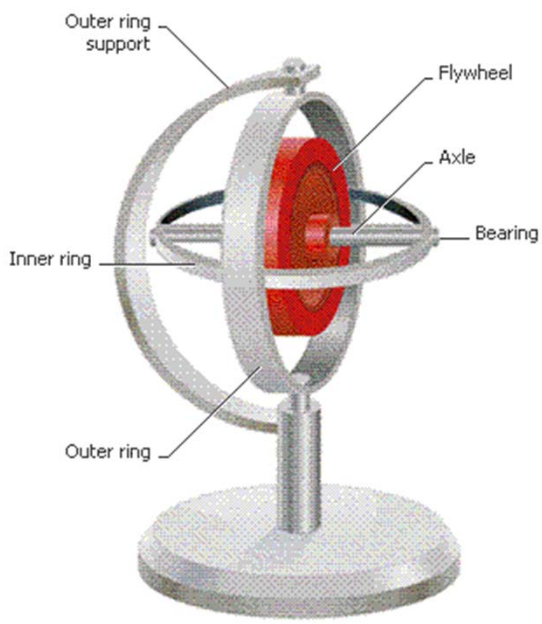


The Gyroscope and Precession

The term gyroscope, refers to any rotating body that exhibits two fundamental properties: gyroscopic inertia, or "rigidity in space", and precession, the tilting of the axis at right angles to any force tending to alter the plane of rotation.

The term gyroscope is commonly applied to spherical, wheel-shaped, or disc-shaped bodies that are universally mounted, so as to be free to rotate in any direction; they are used to demonstrate these properties or to indicate movements in space.

The prefix gyro is customarily added to the name of the application, as, for instance, gyrocompass, gyrostabilizer, and gyropilot.



The rigidity in space of a gyroscope is a consequence of Newton's first law of motion, which states that a body tends to continue in its state of rest or uniform motion unless subject to outside forces. This is also an example of conservation of angular momentum. Thus, the wheel of a gyroscope, when started spinning, tends to continue to rotate in the same plane about the same axis in space.

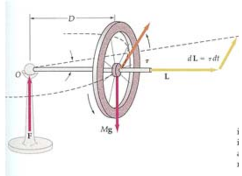
An example of this tendency is a spinning top, which has freedom about two axes in addition to the spinning axis.

Another example is a rifle bullet, which, because it spins in flight, exhibits gyroscopic inertia, tending to maintain a straighter line of flight than it would if not rotating.

Gyroscopes constitute an important part of automatic-navigation or inertial-guidance systems in aircraft, spacecraft, guided missiles, rockets, ships, and submarines. The inertial-guidance instruments in these systems comprise gyroscopes and accelerometers that continuously calculate the exact speed and direction of the craft in motion. These signals are fed into a computer, which records and compensates for course changes.

Precession of a Gyroscope

A simple gyroscope consists of a wheel rotating about an axis as shown below. If the wheel were not rotating, the torque produced by *Mg* would cause the wheel to fall downwards. Note that the directions of all vectors of rotation are given by the right hand rule. Since the wheel is rotating the torque produced by *Mg* must obey the rotational form of

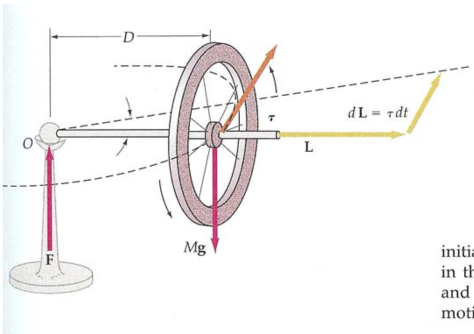


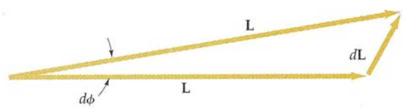
Newton's Law: $\vec{\boldsymbol{\tau}} = \dfrac{d\vec{\mathbf{L}}}{dt} \quad \rightarrow \quad d\vec{\mathbf{L}} = \vec{\boldsymbol{\tau}}dt$

This shows us that the change in angular momentum ($d\vec{\mathbf{L}}$) produced by the torque

$$\vec{\boldsymbol{\tau}} = \vec{\mathbf{r}} \times \vec{\mathbf{F}} \quad \rightarrow \quad \tau = MgD \quad \text{as } \theta = 90°$$

attempting to rotate the wheel downwards is in the direction of the torque, ie. in the horizontal plane. This causes the wheel to rotate about a vertical axis, a motion which is called precession.





When the wheel is spinning with a large initial angular momentum **L** along the axle, the change in the angular momentum $d\mathbf{L}$ is perpendicular to **L**, and the axle moves in the direction of the torque. This motion is called precession.

In the small time interval dt the change in L is

$$dL = \tau \cdot dt = MgD \cdot dt$$

The angle $d\phi = \dfrac{dL}{L} = \dfrac{MgD \cdot dt}{L}$

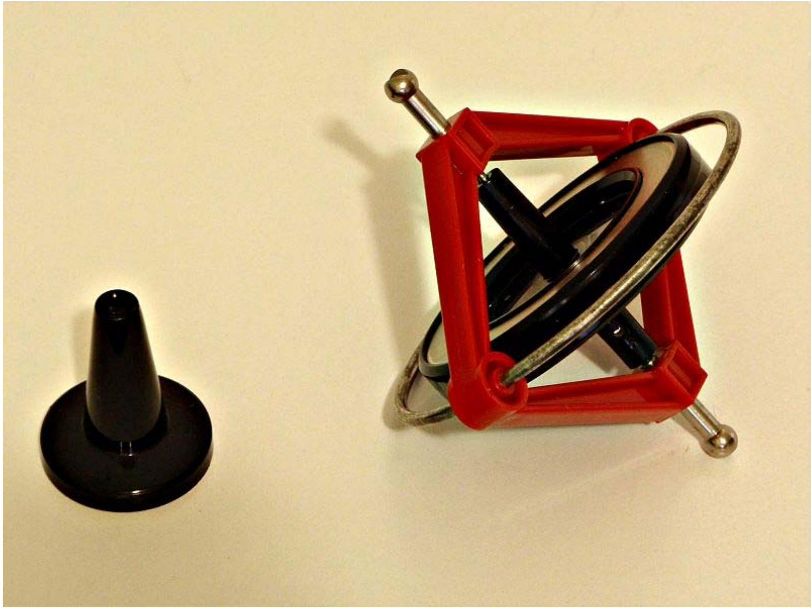
The angular velocity of presession is :

$$\Omega = \frac{d\phi}{dt} = \frac{MgD}{L}$$

Demonstration of Gyroscope and Precession

Click image to show video

A Simple Toy Gyroscope



Demonstration of Gyroscope and Precession

Click images to
show video

Demonstration of Gyroscope and Precession

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Summary of Chapter 11

• Angular momentum of a rigid object:

$$L = I\omega.$$

• Newton's second law:

$$\Sigma\tau = \frac{dL}{dt}.$$

•Angular momentum is conserved.

• Torque:

$$\vec{\boldsymbol{\tau}} = \vec{\mathbf{r}} \times \vec{\mathbf{F}}.$$

Summary of Chapter 11

• Angular momentum of a particle:

$$\vec{\mathbf{L}} = \vec{\mathbf{r}} \times \vec{\mathbf{p}}.$$

• Net torque:

$$\Sigma\vec{\boldsymbol{\tau}} = \frac{d\vec{\mathbf{L}}}{dt}.$$

• If the net torque is zero, the vector angular momentum is conserved.